Assignment 8, due April 20

1. Let $C = S^1 \times [0,1]$ be a cylinder (over a circle). Show that C is a polyhedron. Represent C as a union of two (bent) rectangles with a common boundary consisting of two disjoint segments. Use Mayer-Vietoris sequence to calculate the homology groups of C with integer coefficients.

2. Use the result of the previous problem to find the homology groups $H_k(T^2; \mathbf{Z})$ of the torus $T^2 = S^1 \times S^1$.

3. Find the first homology group (with coefficients in $\mathbf{Z}$) of the surface M_p known from the theorem on classification of compact surfaces (it can be represented as a result of gluing p handles to the sphere with $2p$ holes).

4. What is the second homology group $H_2(M_p; \mathbf{Z})$?

5. Use a simplicial decomposition of the space $\mathbf{RP}^2$ to show that its homology groups $H_k(\mathbf{RP}^2; \mathbf{Z})$ are isomorphic to $\mathbf{Z}, \mathbf{Z_2}, \mathbf{0}$ for $k = 0, 1, 2$ respectively.

6*. Find the homology groups of M_p with coefficients in the group $\mathbf{Z_2}$.