

Assignment 7, due March 22

1. Use the covering of S^1 by \mathbf{R} to obtain another proof that the fundamental group of S^1 is \mathbf{Z} .
2. Find the fundamental group of the real projective space \mathbf{RP}^n .
3. Find the fundamental group of the space $SO(3)$.
4. A covering $p : E \rightarrow B$ is called regular if the image of the fundamental group of E under p_* is a normal subgroup of the fundamental group of B .
 - a) Prove that this definition does not depend on the choice of the base points.
 - b) Prove that any two-fold covering is regular.
- 5*. Prove that a three-fold covering of a two-pretzel (the sphere with two handles, i.e. the surface M_2 in our classification scheme) cannot be regular.
6. Prove that a covering $p : E \rightarrow B$ is regular if and only if for every loop α in the base space, either every path in E covering α is closed or none of them are closed.