

### Assignment 6, due March 14

1. Find the fundamental group of the 3-pretzel, i.e. a sphere with 3 handles.
2. Klein's bottle is defined as a tetragon with the identification pattern described by the word  $aba^{-1}b$ . Find its fundamental group.
- 3\*. Prove that a 2-dimensional surface  $M$  with Euler characteristic  $n$  is homeomorphic to a polygon with  $2n + 4$  edges, identified according to the word:

$$a_1 \dots a_{n+2} a_1^{-1} \dots a_{n+1}^{-1} a_{n+2}^{\pm 1}.$$

That is, first  $n+2$  letters come without inverses, the following  $n+1$  carry inverses and the last one—sometimes yes, sometimes no. Show that it does if and only if  $M$  is orientable.

4. Find the fundamental group of the real projective plane.
5. Describe the fundamental group of the torus.
- 6\*. Prove in detail that the fundamental group of the wedge of  $n$  circles is a free group with  $n$  generators.