Assignment 5, due March 2

1. Let f be a continuous map of the unit interval into itself, such that $f(0) = 0$ and $f(0) = 1$. Show that f is homotopic to the identity map modulo points 0 and 1, i.e. that there exists a homotopy between f and identity which is constant at the endpoints.

2. Let $h : X \to X'$ be a continuous map of topological spaces. For a topological space Y and a continuous map $\alpha : X' \to Y$, let $\Phi(\alpha) = \alpha \circ h$—a map from X to Y. Prove that if α and β are homotopic, then so are $\Phi(\alpha)$ and $\Phi(\beta)$.

3. a) Suppose X is a topological space and A—its path-connected subspace. Prove that if A is a deformation retract of X, then X is path-connected.
b)* Is the conclusion necessarily true if we only assume that X is a retract of A?

4. Prove that the zero-dimensional sphere $\{-1, 1\}$ is not a retract of the one-dimensional disc $[-1, 1]$.

5. Define the Möbius strip M as a square with two opposite sides identified with the same orientation (identifying them with opposite orientations would give a cylinder). Find a (homeomorphic image of) circle C in M which is a deformation retract of M.

6*. Let T be a torus and $x \in T$. Prove that $T \backslash \{x\}$ contains a deformation retract, homeomorphic to a wedge of two circles (a figure "eight").