Assignment 10—extra credit—due May 7

1. Let M, N and P be smooth manifolds; M and N compact and P connected. If $f : M \to N$ and $g : N \to P$ are smooth maps, show that the degree of the composition $g \circ f$ is equal to the product $\deg g \cdot \deg f$.

2. Show that every complex polynomial of degree n extends to a smooth map from S^2 to S^2 of degree n.

3. Let X be a topological space and f, g—two continuous maps from X to S^n, such that for every $x \in X$ $\|f(x) - g(x)\| < 2$. Prove that f and g are homotopic. If, in addition, X is a smooth manifold and f and g are smooth, prove that they are smoothly homotopic.

4. Let X be a compact manifold. Prove that every continuous map $f : X \to S^n$ can be uniformly approximated by smooth maps. Prove that if two smooth maps $f, g : X \to S^n$ are homotopic, then they are smoothly homotopic.

5. Let M be a smooth manifold of dimension m. Prove that if $m < n$, then every smooth map $f : M \to S^n$ is homotopic to a constant.

6. Prove that any smooth map $f : S^n \to S^n$ with degree different from $(-1)^{n+1}$ has a fixed point.