

Mathematical Physics Homework 2, due October 21

0. Please do problems 1.1, 1.2, 1.7 and part 1 of 1.14.

1. Prove that for a bounded operator A , $\sigma(A^*)$ consists of the complex conjugates of the numbers in $\sigma(A)$. Use this to prove that the spectrum of a bounded self-adjoint operator is a subset of \mathbf{R} .

2. Do problem 1.8

3. Let T be an unbounded operator on \mathcal{H}_1 with a domain $D(T)$ and U a unitary operator from \mathcal{H}_1 onto \mathcal{H}_2 . Define an operator S in \mathcal{H}_2 as follows. The domain of S is $U(D(T))$ and for $y \in D(S)$, $Sy = UTU^{-1}y$. We say that S and T are unitarily equivalent. Prove that unitarily equivalent operators have the same spectrum.

4. Use problem 3 to find the spectrum of the (bounded) lattice laplacian, defined as follows. The Hilbert space is $l^2(\mathbf{Z})$, i.e. the space of functions $f : \mathbf{Z} \rightarrow \mathbf{C}$ such that $\sum_{j=-\infty}^{+\infty} |f(j)|^2 < \infty$ and for such f we define

$$(\Delta f)(j) = f(j+1) + f(j-1) - 2f(j).$$

∞ . For extra credit do as many of the problems 1.9 - 1.16 as you can.