Mathematical Physics Homework 1, due September 23

1. Work out Example 2.2 by putting equation (2.11) in a Hamiltonian form, using the function $H(x,p)$ given in (2.13).

2. Prove that symplectic matrices form a group and that if M is symplectic, then:

$$|\det M| = 1; \quad M^{-1} = -JM^TJ; \quad M^T = -JM^{-1}J.$$

Conclude that if M is symplectic then so is M^T.

3. Prove that the Poisson bracket $\{F, G\}$ is antisymmetric in F and G and satisfies Jacobi identity (2.6). Calculate the brackets of the coordinate functions (2.7).

4*. Give an example of a phase space transformation which preserves the form of Hamilton's equation but is not canonical.

5, Do Problem 2.1.

6. Check that, with the definitions (2.38) and (2.39),

$$R(e_3, \theta) = \exp(\theta X_3).$$

7*. Prove that rotations around a unit vector n in $\mathbf{R}^3$ form the flow of the Hamiltonian vector field $J\nabla(L\cdot n)$, where L is the angular momentum vector.

8. Do problem 2.2.