

Mathematics 528A, Homework 2. Due October 21

0. Please do the following problems from Chapter 2: 4, 5, 7, 8, 16, 25*, 29, 41, 75, 101-102.

1. **Unitary invariance of the spectrum.** Prove that if $B = UAU^{-1}$ where U is unitary, then $\sigma(B) = \sigma(A)$.

2*. **Spectrum of the lattice laplacian.** Use problem 1 to find the spectrum of the lattice laplacian, defined as follows. The Hilbert space is $l^2(\mathbf{Z})$, i.e. the space of functions $f : \mathbf{Z} \rightarrow \mathbf{C}$ such that $\sum_{j=-\infty}^{+\infty} |f(j)|^2 < \infty$ and for such f we define

$$(\Delta f)(j) = f(j+1) + f(j-1) - 2f(j).$$

3*. Find the eigenvalues and eigenvectors of Fourier transformation on $L^2(\mathbf{R})$.