Comments to Problem 46, Homework 5

Problem 46, Section 4.5

Show that the boundary value problem

$$y'' + \lambda^2 y = \sin(t) \quad y(0) = 0, \;\; y(\pi) = 1,$$

has a solution if and only if $\lambda \neq \pm 1, \pm 2, \pm 3, \ldots$.

Comments:

As I said in class, there a few cases to consider. If you let $\lambda = \pm 1$, then you should get that

$$y_p = C_1 t\cos(t) + C_2 t\sin(t)$$
$$y_h = C_3\cos(t) + C_4\sin(t)$$

and

$$y = y_h + y_p = C_1 t\cos(t) + C_2 t\sin(t) + C_3\cos(t) + C_4\sin(t).$$

To find C_1 and C_2 you need to plug y_p back into the original equation:

$$y_p'' + y_p = \sin(t).$$

In class I said that to find constants in y_h you need to use the boundary conditions. But notice that **the boundary conditions are given for the function y, and not y_h**:

$$\begin{aligned} y(0) &= y_h(0) + y_p(0) = 0 \\ y(\pi) &= y_h(\pi) + y_p(\pi) = 1. \end{aligned}$$

These boundary conditions **DO NOT** imply that

$$\begin{aligned} y_h(0) &= 0 \\ y_h(\pi) &= 1. \end{aligned}$$

The right way to proceed is to first find C_1 and C_2 by plugging y_p back into the original equation. This should give you

$$y(t) = y_h(t) + y_p(t) = -\frac{t}{2}\cos(t) + C_3\cos(t) + C_4\sin(t).$$

Now use the boundary conditions. First $y(0) = 0$ gives us

$$y(0) = y_h(0) + y_p(0) = C_3 = 0.$$

Then $y(\pi) = 1$,

$$y(\pi) = y_h(\pi) + y_p(\pi) = \frac{\pi}{2} \neq 1$$

which gives us the desired contradiction.

The important thing is to remember that in order to use the boundary conditions to find appropriate constants in your homogenous solution, y_h, first you need to know all the constants in your particular solution, y_p.

See solutions to HW #5 on D2L, for a detailed solution to this problem.