CORRECTION TO "ON THE BRAUER-SIEGEL RATIO FOR ABELIAN VARIETIES OVER FUNCTION FIELDS"

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There is a gap in the proof of Proposition/Definition 4.1 in [Ulm19]. Namely, in the sixth paragraph, the assertion

> Since the break points of a Newton polygon have integer coordinates, $\sum_{\lambda_i<1}(\lambda_i - 1)$ is an integer

is unfounded, because the slopes of the relevant Newton polygon are computed with respect to a valuation v with $v(q) = 1$. If $q > p$, it is not hard to give examples of polynomials whose roots are Weil numbers size q and satisfy a functional equation, yet the relevant break point is not integral. Thus we have not proven in general that $\dim Ш$ is an integer.

It does follow from Remark 4.3(2) of [Ulm19] that $\dim Ш$ is an integer when A is a Jacobian. Moreover, in a recent preprint [LLS$^+$19], T. Suzuki and collaborators have given an Iwasawa-theoretic interpretation of $\dim Ш$ (as suggested in [Ulm19, Remark 4.3(5)]) and proven that it is an integer in general. They also provide an explicit counterexample to integrality of break points.

I thank T. Suzuki for bringing this issue to my attention and for providing a solution.

References

[LLS$^+$19] K.-F. Lai, I. Longhi, T. Suzuki, K.-S. Tan, and F. Trihan, *On the μ-invariants of abelian varieties over function fields of positive characteristic*, 2019. Preprint, arXiv:1909.00511.

[Ulm19] D. Ulmer, *On the Brauer-Siegel ratio for abelian varieties over function fields*, Algebra Number Theory **13** (2019), 1069–1120.

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