CORRECTIONS TO "CURVES AND JACOBIANS OVER FUNCTION FIELDS"

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1. Proposition 4.1

There is a gap in the proof of Proposition 4.1 in [Ulm14] for a general ground field k. The issue is in the penultimate paragraph on page 301, where it is asserted:

> More generally, if $\phi : A \to A'$ is a surjective morphism of abelian varieties over a field k, then we claim that the map of points $A(k) \to A'(k)$ has finite cokernel. If ϕ is an isogeny, then considering the dual isogeny $\phi^\vee$ and the composition $\phi\phi^\vee$ shows that the cokernel is killed by $\deg \phi$, so is finite.

This implicitly assumes that $A'(k)$ is finitely generated. The argument is correct if k is finitely generated (by the Lang-Néron theorem) or if k is algebraically closed (since ϕ is then surjective on points).

This step in the proof is definitely incorrect for some ground fields (such as p-adic fields), and I do not know if the full statement of Proposition 4.1 is correct over a general field k.

Thanks to Cristian Gonzalez Avilés and Kestutis Cesnavicius for pointing out this error.

2. p-adic Tate conjecture

In Section 7.3.1, I state some "straightforward generalizations" of the Tate and Selmer conjectures to the case where the ground field k is finitely generated. Unfortunately, they are false for $\ell = p$. A counterexample is given in Proposition 1.2 of "Boundedness of the p-primary torsion of the Brauer group of an abelian variety" by Marco D'Addezio (Compositio Mathematica **160**, (2024), 463–480).

Also, the exact sequences in the first display on p. 305 and the second display on p. 321 are not properly justified. (The issue is that $\mathrm{NS}(\overline{\mathcal{X}}) \otimes \mathbb{Z}_\ell$ should be given the ℓ-adic topology.) Assuming semisimplicity (Tate's S^1) suffices to fix this issue, and it is known that $T^1 \implies S^1$ for $\ell \neq p$.

Thanks to Marco D'Addezio for pointing out these issues.

References

[Ulm14] D. Ulmer, *Curves and Jacobians over function fields*, Arithmetic geometry over global function fields, 2014, pp. 281–337.

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