Solutions For Math 111 Test 2 Summer 2012

(Use graphic calculator or wolfram to check graphs.)

1. $y = -1 + 3\sin(2x-1)$. We sketch the following graphs:
   - $0 \le 2x - 1 \le 2\pi \implies \frac{1}{2} \le x \le \pi + \frac{1}{2}$ so sketch "standard sine shape" inside $[\frac{1}{2}, \pi + \frac{1}{2}]$. This is $y_0 = \sin(2x-1)$.
   - Change amplitude to 3, i.e. the max and min is now 3 and -3 respectively. This is $y_1 = 3\sin(2x-1)$.
   - Move the whole graph down by 1 unit. This is $y = -1 + 3\sin(2x-1)$.
   - Sketch another cycle of the graph next to the above.
2. (a) $\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x) = 2\sin(x)\cos(x)$.
   (b) $y = \sin(x)\cos(x) = \frac{1}{2}\sin(2x)$. $0 \le 2x \le 2\pi \implies 0 \le x \le \pi$, so sketch one sine cycle in the interval $[0, \pi]$. Change the amplitude to $\frac{1}{2}$. Sketch another cycle next to the graph.
3. (a) $-\frac{\pi}{2} < x - \frac{\pi}{2} < \frac{\pi}{2} \implies 0 < x < \pi$. Sketch "standard tangent shape" in $(0, \pi)$.
   (b) Asymptotes are given by the line equation $x = \pi \cdot n$ for all $n \in \mathbb{Z}$.
   (c) Zeroes are at $x = \frac{(2k+1)\pi}{2} \cdot \pi$, $k \in \mathbb{Z}$. (odd number times π divided by 2).
4. (a) $\cot(\frac{\pi}{4})\sin(\frac{\pi}{2}) - \cos(\frac{\pi}{2}) = 1 \cdot 1 - 0 = 1$.
   (b) LHS$= \cot(\theta)\sin(2\theta) = \dfrac{\cos(\theta)}{\cancel{\sin(\theta)}} \cdot 2\cancel{\sin(\theta)}\cos(\theta) = 2\cos^2(\theta) =$RHS,
   where the last inequality is given by double angle identity of cosine.
5. (a) A periodic function is a function $f(x)$ which satisfies the property $f(x) = f(x + np)$ for all $n \in \mathbb{Z}$ and a fixed p, the period. p is the unique smallest positive number satisfying the equality for all $n \in \mathbb{Z}$. This means the graph of f repeats itself after any interval of length p on the x-axis.
   (b) $f(x) = \sin(2\pi x)$.
   (c) $f(x) = \sin(\pi x)$.
6. Note the RHS is an expression purely in terms of $\cos(x)$ so you should try to eliminate any $\sin(x)$ that appears when applying IDs:

$$\begin{aligned}
\text{LHS} &= \cos(2x + x) = \cos(2x)\cos(x) - \sin(2x)\sin(x) \\
&= (2\cos^2(x) - 1)\cos(x) - 2\sin(x)\cos(x)\sin(x) \\
&= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x) \\
&= 2\cos^3(x) - \cos(x) - 2(1 - \cos^2(x))\cos(x) \\
&= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x) \\
&= 4\cos^3(x) - 3\cos(x) = \text{RHS}
\end{aligned}$$

7. (a) $f(70) = f(62) = f(54) = \ldots = f(14) = f(6) = f(-2)$ so is impossible.
   (b) By periodicty, the graph between $[0, 8]$ and $[8, 16]$ should looks exactly the same. e.g. the line $y = x$ from 0 to 8, and the repeat the same graph from 8 to 16.
   (c) $a = 8,000,001$ works. Basically any number n satisfying $\frac{n-1}{8} \in \mathbb{Z}$.
8. (a) $f(x) = f(y) \implies x = y$. Passes horizontal line test.
   (b) Periodic $\implies f(x) = f(x+p)$ but $x \neq x + p$.
   (c) $f(x) = x^2$ not one-to-one since $f(1) = f(-1)$ but $1 \neq -1$. Can make it periodic by restricting $x \ge 0$ or $x \le 0$.
9. $\cos(-\frac{25\pi}{12}) = \cos(\frac{25\pi}{12}) = \cos(\frac{\pi}{12}) = \cos(\frac{\pi}{3} - \frac{\pi}{4}) = \cos(\frac{\pi}{3})\cos(\frac{\pi}{4}) + \sin(\frac{\pi}{3})\sin(\frac{\pi}{4}) = \frac{1+\sqrt{3}}{2\sqrt{2}}$
10. (a) $\cos(2\theta) = 2\cos^2(\theta) - 1 = \frac{2}{16} - 1 = -\frac{7}{8}$.
    (b) 2θ must be in QII so $45° < \theta < 90°$.