Homework Section 4.0 - Due 10th October

1. Suppose $f(x)$ is a periodic function with $p = 8$.
   Suppose further that $f(1) = 4$, $f(0) = -6$, $f(-2) = -2$. If possible, compute
   (a) $f(-20)$.
   (b) $f(56)$.

2. Suppose $f(x)$ is a periodic function with $p = 7$ and $f(85) = 13$, $f(214) = 12$, $f(38619) = 11$. If possible, compute
   (a) $f(0)$.
   (b) $f(1)$.
   (c) $f(2)$.

3. Sketch the graph of

$$f(x) = \begin{cases} 2 & \text{if} \quad x \in [2k, 2k+1) \\ -3 & \text{if} \quad x \in [2k-1, 2k) \end{cases} \qquad \text{for all } k \in \mathbb{Z}$$

   over the interval $[-5, 5)$.
   Is this function periodic? If so, what is its period?

4. Mark the following true or false. If true, explain why, if false, give a counterexample:
   (a) x_1 and x_2 are coterminal angles, then $\cos(x_1) = \cos(x_2)$.
   (b) If $\cos(x_1) = \cos(x_2)$, then $x_1 = x_2$.
   (c) If t and s have the same reference angle, then $\cos(t) = \cos(s)$.
   (d) If $\sin(x_1) = \sin(x_2)$, then their reference angles $x_1' = x_2'$.
   (e) If $f(x)$ is a periodic function with $f(x) = 0$ for every integer x, then the period of $f(x)$ must be 1.

5. Functions are not integers.
   (a) Give an explicit example of function that is neither even nor odd.
   (b) Give an explicit example of function that is both even and odd.

*6. The <u>amplitude</u> of a periodic function $f(x)$ is given by $a = \frac{|\max(f(x)) - \min(f(x))|}{2}$.
   Calculate the amplitude of
   (a) $f(x)$ as in question 3.
   (b) $f(x) = \sin(x)$.
   (c) $f(x) = \cos(x)$.

   Is it possible to have a function with amplitude of -1?
   If possible, give an example of such function, if impossible, explain why.