Hodge structure

$\Delta \subset M_{\mathbb{R}}$

$$P_\Delta(t) = \sum_{k\geq 0} \ell(k\Delta)\, t^k$$

$$\ell(k\Delta) = \#(M \cap k\Delta).$$

$$S_\Delta = \bigoplus_{k\geq 0} S_\Delta^k$$

$$P_\Delta(t) = \frac{\Psi_\Delta(t)}{(1-t)^{d+1}}$$

$$\deg \Psi_\Delta(t) \leq d$$

$d = 1$

$$\ell(k\Delta) = kn+1$$

$$P_\Delta(t) = \sum_{k\geq 0} (kn+1)\, t^k =$$

$$= \frac{1}{1-t} + \frac{nt}{(1-t)^2} = \frac{1+(n-1)t}{(1-t)^2}$$

?

$$Q_\Delta(t) = \sum_{k>0} \ell^*(k\Delta)\, t^k$$

$$\ell^*(k\Delta) = \#\left(\mathrm{Int}(k\Delta) \cap M\right)$$

$$Q_\Delta(t) = \frac{\varphi_\Delta(t)}{(1-t)^{d+1}}$$

$$\deg \varphi_\Delta(t) = d+1$$

Ex.

$$Q_\Delta(t) = \sum_{k>0} (kn-1)\, t^k = \frac{t^2+(n-1)t}{(1-t)^2}$$

$$\varphi_\Delta(t),\ \psi_\Delta(t)$$

$$\varphi_\Delta(t) = t^{d+1}\,\psi_\Delta(t^{-1})$$

$$\varphi_\Delta(t) = \sum \varphi_i(\Delta)\, t^i$$

$$\psi_\Delta(t) = \sum \psi_j(\Delta)\, t^j$$

$\cancel{\varphi}$

$\psi_i(\Delta) = \varphi_{d+1-i}(\Delta)$

$\forall i \quad 0 \le i \le d$

$\psi_0(\Delta) = 1 \quad \varphi_1(\Delta) = \ell^*(\Delta)$

$\psi_d(\Delta) \quad \varphi_{d+1}(\Delta)$

Behind

$S_f = S_\Delta / \langle F_0, F_1, \dots F_d \rangle S_\Delta$

$P(S_f, t) = \psi_\Delta(t)$

$\dim S_f^i = \psi_i(\Delta)$

$I_\Delta \subset S_\Delta$ canonical ideal.

$\langle \mathrm{Int}\, C_\Delta \cap M \rangle$

$I_\Delta / \langle F_0, F_1, \dots F_d \rangle I_\Delta = I_f$

$Q_\Delta(t) \longleftarrow I_\Delta$

$\varphi_\Delta(t) = (1-t)^{d+1} Q_\Delta(t)$

$\dim I_f^k = \varphi_k(\Delta)$

$S_f \ , \ I_f$

$S_f \times I_f \to I_f$

$S_f^i \times I_f^{d+1-i} \to I_f^{d+1} \cong K$ ④

$S_f^i \cong \left(I_f^{d+1-i}\right)^*$.

Kohomology:

$H^i(Z_f), H^i_c(Z_f)$

$Z_f \subset \mathbb{T}^d \qquad d \geq 2$

$H^0(Z_f) = \mathbb{Z}$

$$H_c^{2(d-1)}(Z_f) \cong \mathbb{Z}$$

$$Z_f \subseteq \mathbb{T}^{d-1}(\mathbb{C}) \cong (\mathbb{C}^*)^{d-1}$$

$$H^i(Z_f) \times H_c^{2d-2-i}(Z_f) \to$$
$$\to H_c^{2(d-1)}(Z_f) \cong \mathbb{Z}$$

$$S_f \times I_f \to I_f .$$

$$Z_f \subset (\mathbb{C}^*)^d$$

$$\sum_i (-1)^i \dim H^i(Z_f) =$$
$$= (-1)^{d-1} \underbrace{\mathrm{vol}(\Delta)}_{= \, d! \times (\text{volume } \Delta)}$$

$$\dim S_f = \dim I_f = \underline{\mathrm{vol}(\Delta)}$$

6

Lefschetz Th.

for $Z_f \subset \mathbb{T}^d$.

$H^i(\mathbb{T}^d) \to H^i(Z_f)$

iso. $0 \le i \le d-2$

inj $i = d-1$

Z_f affine

$H^i(Z_f) = 0 \quad i \ge d$.

$H^{d-1}(Z_f)$

$\dim H^{d-1}(Z_f) =$

$= vol(\Delta) + d - 1$

$H^{d-1}(\mathbb{T}^d) \hookrightarrow H^{d-1}(Z_f)$

$\dim PH^{d-1}(Z_f) =$

$= vol(\Delta) - 1$

$H^i(X) \quad H^i_c(X)$

Deligne.

Hodge filt.

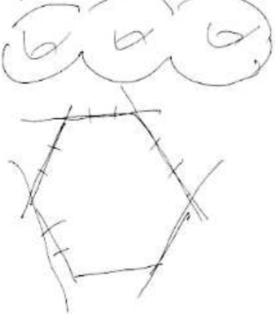
Weight filt.

$H^{p,q}(X)$

$d=2 \qquad Z_f \subset (\mathbb{C}^*)^2$ ⑦

affine curve

$\overline{Z_f}$ smooth projective



⑧

$\#(\overline{Z}_f \setminus Z_f) = \# \partial\Delta \cap M$

$H^i(\overline{Z}_f)$

0	1
1	2g
2	1

$e(\overline{Z}_f) = 2-2g$

$\overline{Z}_f$

$H^2(Z_f) = 0$

$\dim H^1(Z_f) = \dim H^1(\overline{Z}_f) +$
$+ (\# \partial\Delta \cap M) - 1)$

$\dim H^0(Z_f) = 1$

$$\dim H^{d-1}(Z_f) = \mathrm{Vol}(\Delta) + d - 1$$

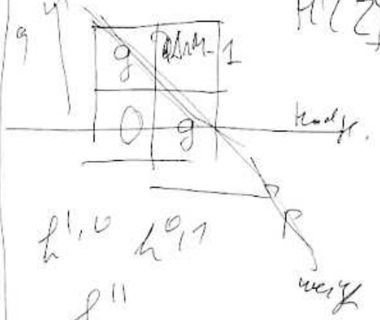
$d = 2$

$$\dim H^1(Z_f) = \mathrm{Vol}(\Delta) + 1$$

$$2g + \#(\partial\Delta \cap M) - 1$$

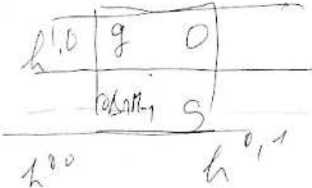
$$g = \#\ell^*(\Delta)$$

$H^1(Z_f^{(g)})$



$h^{1,0}$ $h^{0,1}$

h^{11}



$\mathcal{H}^1(Z_f)$

$h^{1,0} + h^{1,1} = \ell^*(\Delta)$

$\text{for } \Delta \cap M \to$

$= \#(\Delta \cap M) - 1 = S_f^1$

$\ell(\Delta) - d - 1$

-2

$h^{0,0} + h^{0,1} = \dim S_f^2$

$h^{0,0} = 0$, $h^{0,1} = \ell^*(\Delta)$

$\forall \Delta \quad d.$

$$Gr_H PH^{d-1}(Z_f) \cong S_f^+ \subset S_f$$

$$Gr_H PH_c^{d-1}(Z_f) \cong$$

$$\cong \bigoplus_{i=1}^{d} I_f$$

General statement

$$H^*(Z_f)$$

$$H^*(\mathbb{T}^d) \cong \Lambda^* M$$

Koszul complex

$F_0, F_1, \dots, F_d$

in S_Δ

Twisted version

$$M_{\mathbb{R}} \supset \Delta$$

$$\check{M} \subset M' \subset M_{\mathbb{Q}}$$

$$f(x) = \sum_{m \in A} a_m x^m$$

$$Z'_{f,M'} \to Z_{f,M} \quad [M' : M]$$

Secondary polytope

1

Ex: $A=[0,1,\ldots,n]$

$\Delta=[0,n]$

$$f(x)=\sum_{i=0}^{n} a_i x^i$$

$a_0 a_n D(f)$

↑ discriminant.

$$\begin{array}{c} n-1\left\{\begin{array}{c} \\ \\ \end{array}\right. \\ n\left\{\begin{array}{c} \\ \\ \end{array}\right. \end{array}\left|\begin{array}{cccccc} a_0 & a_1 & \ldots & a_n & & \\ & a_0 & a_1 & & a_n & \ldots \\ a_1 & 2a_2 & \ldots & n a_n & & \\ & a_1 & 2a_2 & \ldots & & \end{array}\right|$$

$(2n-1)\times(2n-1)$

$\det C$.

$F_0, \ldots, F_d \in S_\Delta^1 \subset S_\Delta$

$$S_f := S_\Delta / \langle F_0, \ldots F_d \rangle S_\Delta$$

$$f = \sum_{m \in A} a_m x^m$$

$R = \cancel{\mathbb{Z}}[a_1, \ldots a_n] \qquad n = |A|$

$$S_f \leftarrow S_\Delta \leftarrow S_\Delta(-1) \leftarrow \ldots$$

$$S_f^k \leftarrow S_\Delta^k \leftarrow \Lambda^1 \tilde{N} \otimes_{\mathbb{Z}} S_\Delta^{k-1} \leftarrow \ldots$$

$k \gg 0$

$\det C_\cdot^k(f)$

Th. indep. on $k \gg 0$

$\det C_\cdot^k(f) \in R$

$\| $

$E_A(f)$

principal A-determinant of f.

$A = \{0, \ldots, n\}$ (2)

$E_A(f) = \pm a_0 a_n D(f)$

$E_A(f) = ?$

$$E_A(f) \overset{\text{"}}{=} \prod_{\Theta \subset \Delta}{}'' \underline{\mathcal{D}}''(f_\Theta)$$

Secondary polytope.

Newton polytope of

$E_A(f)$ $\quad n = |A|$

$\sum_{\neq 0} {}'' c_\mu a^\mu \quad a_1 \ldots a_n$

$\mathbb{Z}^n \supset L$ codim $d+1$

$Sec(A) = Conv(\mu)$

Gelfand-Kapranov-Zelevinsk

all vertices of $Sec(A)$

$\forall A$

$d = 1$

Newton diagram of

1 dim def. of f

$f_t(x) = t^{m_0} + t^{m_1} x + \ldots + t^{m_n} x^n$

$m_0, m_1 \ldots m_n \in \mathbb{Z}_{>0}$

$\underline{d=1}$ $\Delta = [0, n]$

$D(f) = a_n^{2n-2} \prod_{i<j} (\rho_i - \rho_j)^2$

$\rho_1, \dots, \rho_n$ roots of f.

$\mathbb{T}^1 \times f$

$(\lambda \cdot f) \Rightarrow$

$= a_0 + \lambda a_1 x + \lambda^2 a_2 x^2 + \dots + \lambda^n a_n x^n$

$wt(a_i) = i$

$D(f)$ quasi-hom. of degree $n^2 - n$

(3)

$D(f) = D(a_0, a_1, \dots, a_n)$

$\underset{\neq 0}{c_k} \, a_0^{k_0} a_1^{k_1} \dots a_n^{k_n}$

$0 \cdot k_0 + 1 \cdot k_1 + \dots + n k_n = n^2 - n$

All monomials in $D(f)$

$D(f) \longrightarrow E_A(f) = \underline{a_0 a_n} D(f)$

$(2n-2, n^2-n)$ $(2n, n^2)$

$$D(f_t) = c\, t^{\varepsilon}(1+o(t))$$

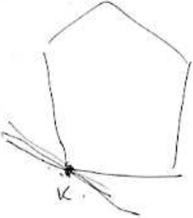
$t \to 0$

$$D(t^{m_0}, \dots, t^{m_n}) \qquad a_i \to t^{m_i}$$

$m = (m_0, \dots, m_n)$

$$= \sum c_k t^{\langle k, m \rangle}$$

$\sum m_i y_i$

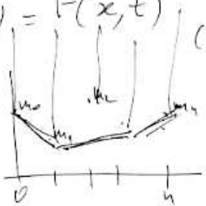


$$D(f) = a_n^{2n-2} \prod_{i<j} (\rho_i - \rho_j)^2$$

$$\rho_i = c_i\, t^{\varepsilon_i}(1 + o(1))$$

Newton diagram of

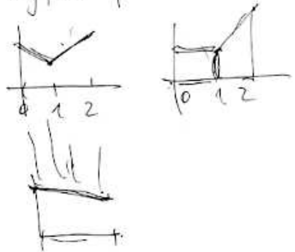
$f_t(x) = F(x,t)$ (i, m_i)



Vertices of $\mathrm{Sec}(\Delta(f))$

$\Updownarrow$

types of Newton diagrams

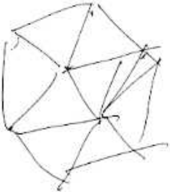


$I \subset \{1, \dots, n-1\}$

2^{n-1} vertices of cube $\dim = n-1$

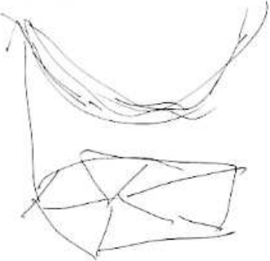
GKZ

Δ



$$f_t(x) = \sum_{m \in A} \underbrace{t^{\boxed{\varphi(m)}}}_{a_m} x^m$$

$\varphi: A \to \mathbb{Z}_{>0}$

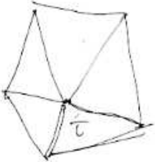


⑦

Monomials in $E_A(f)$

$\uparrow$

vertices

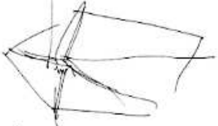


$$\boxed{\prod_{\dim \tau = d} E_\tau(a)}$$

$$\prod_{\dim \tau = d} \mathrm{vol}(\tau)^{\mathrm{Vol}(\tau)}$$

$$E_\tau(a) = (\mathrm{Vol}\,\tau)^{\mathrm{Vol}(\tau)}(a_0 \dots a_d)^{\mathrm{Vol}(\tau)}$$

$\varphi(m)$



$$T^d \times E_A(f) = E_A(f)$$

$$\boxed{(d+1)\,d!}$$

$\Delta \ni \beta.$

Ex Δ simplex

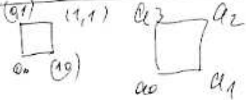
A = vertices of Δ

$$f = \sum_{m \in A} a_m x^m$$

$$E_A(f) = \pm (\mathrm{Vol}(\Delta))^{\mathrm{Vol}(\Delta)} (a_0 \dots a_d)^{\mathrm{Vol}(\Delta)}$$

$\mathrm{Vol}(\Delta) = d!\,(\text{volume } \Delta)$

Ex:



$$E_A(f) = a_0 a_1 a_2 a_3 \times (a_1 a_3 - a_0 a_2)$$

$E_A(f)$ homogeneous of degree $(d+1)\mathrm{Vol}(\Delta)$

$d = 1 \quad \Delta = [0, n]$

degree $E_A(f) = 2n$

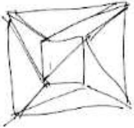
degree $D(f) = 2n - 2$.

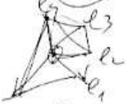
d more homogeneity condition, $\mathbb{T}^d$.

$|A| = d+2$

5^5

$1 \cdot \; -5^5$





$e_5 = -\sum e_i$

①

A-hypergeometric system

$A \subset M \simeq \mathbb{Z}^d$

$\langle A, u \rangle = 1 \quad \forall u \in N$

regular case.

$f(x) = \sum_{m \in A} a_m x^m$

$\Psi_\beta(a) = \int_C x^{-\beta} \exp(f(x)) \frac{dx}{x}$

$C \quad \beta \in M_{\mathbb{C}}$

$C \in T^d(\mathbb{C})$

$I(\beta, a)$

$A = \{v_1, \dots, v_n\}$

$R(A) \to \mathbb{Z}^n \to M$

$(.\,0\,1\,0) \to v_i$

$\to \mathbb{R}^n \to M_{\mathbb{R}} \to 0$

$R(A) = (\lambda_1 \dots \lambda_n) \in \mathbb{Z}^n$

$\sum \lambda_i v_i = 0$

$n-d$

②

$\cancel{\lambda} \doteq (\lambda_1, \dots, \lambda_n) \in R(A)$

$$\prod_{\lambda_i<0} \left(\frac{\partial}{\partial a_i}\right)^{-\lambda_i} I(\beta, a) = \prod_{\lambda_i>0} \left(\frac{\partial}{\partial a_i}\right)^{\lambda_i} I(\beta, a)$$

$\cancel{(1, \dots, 1)}$

$$\Box_\lambda = \prod_{\lambda_i<0} - \prod_{\lambda_i>0}$$

$$\Box_\lambda I(\beta, a) = 0$$

$\lambda \in R(A)$

$\beta \in M_{\mathbb{C}}$ $\quad \beta \in M_{\mathbb{Q}}$

Choose a Basis of N

$u_1, \dots, u_d$ $\quad 1 \le k \le d$

$$D_k^\beta = \sum_{i=1}^{n} \langle v_i, u_k \rangle a_i \frac{\partial}{\partial a_i} - \langle \beta, u_k \rangle$$

$$D_k^\beta I(\beta, a) = 0$$

(3)

$\Box_\lambda \varphi(a) = 0$

$\lambda \in R(A)$

$D_i^\beta \varphi(a) = 0 \quad 1 \le i \le d$

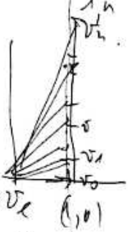
A - hyperplan. system.

$\underbrace{\{0, 1, \dots n\}}_{A} \in \mathbb{Z} = M$

$a_0, a_1, \dots a_n$

$f(x) = \sum_{i=0}^{n} a_i x^i$

$\tilde{M} = \mathbb{Z} \oplus M$



$v_i + v_j = v_k + v_l$

iff $i + j = k + l$

4

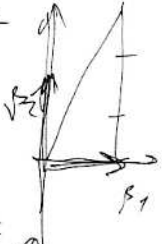
$$v_i + v_j - (v_k + v_l) = 0$$

$$\left(\frac{\partial}{\partial a_i}\frac{\partial}{\partial a_j} - \frac{\partial}{\partial a_k}\frac{\partial}{\partial a_l}\right)\varphi(a) = 0$$

$$i + j = k + l.$$

$$\beta \in \tilde{M} = \mathbb{Z}^2$$

$$\beta = (\beta_1, \beta_2)$$



$$\left(\sum_{i=0}^{n} a_i \frac{\partial}{a_i}\right)\varphi = \beta_1 \varphi.$$

$$\left(\sum_{i=0}^{n} i\, a_i \frac{\partial}{\partial a_i}\right)\varphi = \beta_2 \varphi$$

$$Z_f \subset \mathbb{T}^d \quad f = 0$$

$$H_{d-1}(Z_f, \mathbb{Z}) \ni \gamma$$

$$\omega \in H^{d-1}(Z_f)$$

$$\int_\gamma \omega =$$

$$\Omega^{d-1} Z_f \xleftarrow{\text{res.}} \Omega^{d}(T^{d} \setminus Z_f)$$

$$\frac{x^n}{(f)^n}\frac{dx}{x}$$

$$f(x)=\sum a_i x^i$$

$Z_f \subset \mathbb{C}^*$ roots of $f(x)$

x

Mayer

$\rho_1^\mu, \dots, \rho_n^\mu$

solutions to

A-hyperge. system.

$\beta=(0,-\mu)$

$n=2$

$R(A)$

$(1,-2,1)$ $\quad \frac{\partial}{\partial a_0}\frac{\partial}{\partial a_2}-\left(\frac{\partial}{\partial a_1}\right)^2$

β

$a_0 + a_1 x + a_2 x^2$

$\sqrt{a_1^2 - 4a_0a_2}$

$a_1^2 \gg 4a_0a_2$

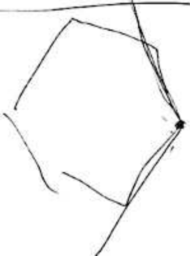
$a_1\sqrt{1 - \frac{4a_0a_2}{a_1^2}}$

$z = \frac{a_0a_2}{a_1^2}$ $\sqrt{1-4z}$

$4a_0a_2 \gg a_1^2$

$\sqrt{-4a_0a_2}\,\sqrt{1 - \frac{a_1^2}{4a_0a_2}}$

$w = \frac{a_1}{a_0a_2}$ $\sqrt{1 - \frac{\cancel{4}w}{4}}$



Sec.

$n=2$

$a_1^2 \quad -2^2 a_0 a_2$

$n=3$

See $\sim$ 2^{n-1}-dim. cube.

2^{n-1}

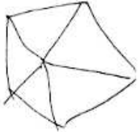
Sturmfels.

A regular case.

for generic β

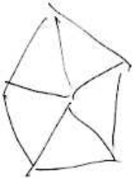
$\mathrm{Conv}(A)=\Delta \quad \dim \Delta = d-1$

T convex triangulation.



$d=3$

⑧



Δ

nonresonant β

$\mathrm{Vol}(\Delta)$

$$\boxed{\Phi_\alpha(a) = \sum_{\ell \in R(A)} \prod_{i=1}^{n} \Gamma(\alpha_i + \ell_i + 1)^{-1} a^{\ell+\alpha}}$$

$\lim \mathrm{Sec}(A) = |A| - d$

$$\alpha \in \mathbb{C}^n \longrightarrow M_{\mathbb{C}} \ni \beta \qquad 0 \to R(A)_{\mathbb{C}} \to \mathbb{C}^n \to M_{\mathbb{C}} \to$$

$\underline{\text{cg}}$ $\Theta(x_0 f)$

$n=2$ $(1,-2,1)$

$d=5$

$(1,1,1,1,1,-5)$

⑨

$f(x)=0$

$f(x)=\underline{\underline{t}} \quad t\to\infty$

$\Delta_\infty=\{0,A\}$

$$J_0(x)=\frac{1}{2\pi i}\int_C e^{\left(z-\frac{x^2}{4z}\right)}\frac{dz}{z}$$

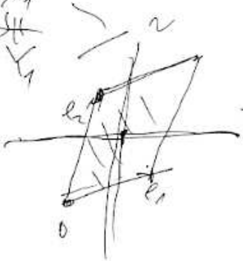
Toric mirror symmetry

X CY — $N=2$ SCFT

Y CY —

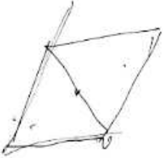
Example. (elem. geometry)

$X_1 \not\cong Y_1$

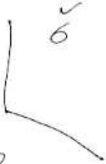


$\frac{1}{2}$ twists.

$GL(2,\mathbb{Z}) \ni \varphi$

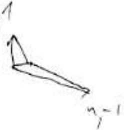


$M_{\mathbb{R}}$



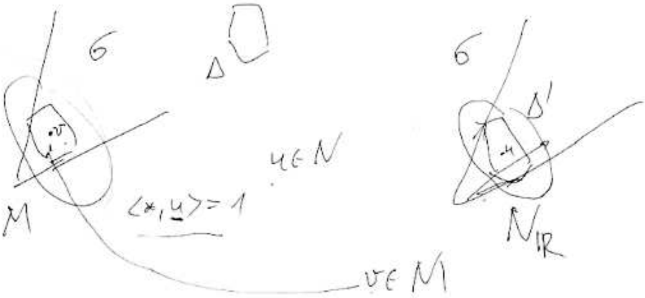
$N_{\mathbb{R}}$

$\check{\sigma} = \{y : \langle x, y \rangle \geq 0 \ \ \forall x \in \sigma\}$.



$n \neq 1, 2$

Toric mirror symmetry.



$\langle v, x \rangle = 1$

reflexive polytope

$\langle u, u \rangle = 1$

$$Z_f \subset \mathbb{T}^2 \simeq (\mathbb{C}^*)^2$$ (4)

affine curve

$$\left.\begin{array}{l} \dim H^1(Z_f) = Vol(\Delta)+1 \\ \dim PH^1(Z_f) = Vol(\Delta)-1 \end{array}\right\} -2 = \dim T^2$$

$$\boxed{\begin{array}{c|c} \ell^*(\Delta) & \#(\partial\Delta\cap M)-3 \\ \hline 0 & \ell^*(\Delta) \end{array}} \begin{array}{l} = \ell(\Delta)-3 \\ = \ell^*(\Delta) \end{array}$$

$$PH^1(Z_f) \underset{?}{\simeq} \cancel{H^2(X(p))\oplus H} \; \bigoplus_{i>0} H^i(X(p))$$

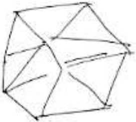
only filtration

~~doesn't depend~~

graded / depend on triangulation.

(5)

<u>Ex</u> $d=3$ $\quad \dim \Delta = 2$



$\exists$ unimodular convex triangulation for any Δ.

$$H^0(X(p)) = \mathbb{Z}, \quad H^2(X(p)) = \mathbb{Z}^{\ell(\Delta)-3}$$

$$H^4(X(p)) = \mathbb{Z}^{\ell^*(\Delta)}$$

$$\Psi_\Delta(t) = \cancel{\Psi_0} + \psi_1 t + \psi_2 t^2 = (1-t)^3 \sum_{k \geq 0} \ell(k\Delta) t^k$$

$$\varphi_\Delta(t) = \underset{\substack{\| \\ \ell^*(\Delta)}}{\varphi_1} t + \varphi_2 t^2 + \underset{\substack{\| \\ 1}}{\varphi_3} t^3 = (1-t)^3 \sum_{k \geq 0} \ell^*(k\Delta) t^k.$$

$$\boxed{1 + \ell(\Delta) - 3 + \ell^*(\Delta) = \mathrm{Vol}(\Delta)}$$

Pick's ~~theorem~~

⑥

$$\begin{array}{cccc} \psi_0(\Delta) & \psi_1(\Delta) & \dots & \psi_{d-1}(\Delta) \\ \parallel & & & \\ \wedge & & & \\ \varphi_1(\Delta), & \varphi_2(\Delta) & \dots & \varphi_d(\Delta) \\ & & & \parallel \\ & & & 1 \end{array}$$

<u>Th.</u> If $X(p)$ is smooth, then $X(p)$

$$\begin{array}{cccccc} H^0(X,p) & H^1 & H^2 & & H^{2(d-1)} \\ \parallel & \parallel & & & \parallel \\ \mathbb{Z} & 0 & \mathbb{Z}^{\psi_1(\Delta)} & & \mathbb{Z}^{\psi_{d-1}(\Delta)} \end{array}$$

$\dim X(p) = d$, but $H^{2d} = 0$

$X(p)$ not compact.

$$\begin{array}{ccc} H^0_c(X,\mathbb{Z}) & H^2_c(X,\mathbb{Z}) & \dots \\ \parallel & \parallel & \\ 0 & \mathbb{Z}^{\varphi_1(\Delta)} & \end{array}$$

$H^k(X(p))$	0	1	2	3	
$\dim H^i$	$\cancel{[illegible]}$	0	4	$\cancel{[illegible]}$	4

(7)

$$0 \to R(A)_{\mathbb{R}} \to \mathbb{R}^n \to M_{\mathbb{R}} \to 0$$

$$0 \to N_{\mathbb{R}} \to \mathbb{R}^n \xrightarrow{\alpha} R(A)^*_{\mathbb{R}} \to 0$$

$$\mathbb{C}^n \to \mathbb{R}^n$$

$$(z_1, \dots, z_n) \to (|z_1|^2, \dots, |z_n|^2)$$

$$\mu_A : \mathbb{C}^n \to \mathbb{R}^n \xrightarrow{\alpha} R(A)^*_{\mathbb{R}}$$

$$H(z) = \sum_{i=1}^{n} |z_i|^2 \text{ on } \mathbb{C}^n$$

$\mu_A^{-1}(p)$ smooth $p \in \mathrm{Int}\ \sigma \in \Sigma(A)^{(n-d)}$

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$$f(x) = \sum_{m \in A} a_m x^m$$

$$Z_f \subset (\mathbb{C}^*)^d$$

$$\dim PH^{d-1}(Z_f) = \mathrm{Vol}(\Delta) - 1 = d(\Delta) - 1$$

Hodge ~~Zahlen~~ Vektr.

$$S_\Delta \rightsquigarrow P_\Delta(t) = \sum_{k \geq 0} \ell(k\Delta) t^k =$$

$$= \frac{\overset{1}{\varphi_0}(\Delta) + \varphi_1(\Delta) t + \ldots + \varphi_d(\Delta) t^d}{(1-t)^{d+1}} \qquad S_f = S_\Delta / (F_0 \ldots F_d)$$

$$\varphi_i = \dim S_f^i$$

$$I_\Delta \to Q_\Delta(t) = \sum_{k \geq 0} \ell^*(k\Delta) t^k$$

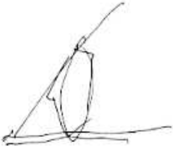
$$Q_\Delta(t) = \frac{\varphi_\Delta(t)}{(1-t)^{d+1}} \longleftrightarrow I_f = I_\Delta / \langle F_0 \ldots F_d \rangle$$

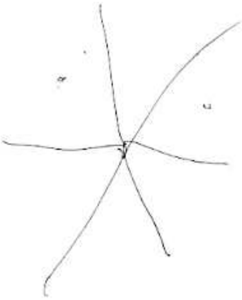
$$S_f^i \times I_f^{d+1-i} \to I_f^{d+1} \cong \mathbb{C}.$$

$$\Delta \to (1, \Delta)$$

$$M_\mathbb{R} \qquad \tilde{M}_\mathbb{R}$$

$\tilde{M}$ lattice of rank $d+1$





Secondary
polytope
and fan.

$|A| = n$

$\widetilde{M} \ni (1, v_i) \quad i = 1, \ldots n$

$$0 \to R(A) \to \mathbb{Z}^n \to \widetilde{M}$$

$$e_i \longrightarrow (1, v_i)$$

$$0 \to R(A)_{\mathbb{R}} \to \mathbb{R}^n \to \widetilde{M}_{\mathbb{R}} \to 0$$

$\dim R(A)_{\mathbb{R}} = n - d - 1$

$$0 \to \widetilde{N}_{\mathbb{R}} \to \mathbb{R}^n \to R(A)^*_{\mathbb{R}} \to 0$$

$p \in R(A)^*_{\mathbb{R}}$

$\mu : \mathbb{C}^n \to \mathbb{R}^n \xrightarrow{\alpha} R(A)^*_{\mathbb{R}} \to 0$

$(z_1, \dots, z_n) \to (|z_1|^2, \dots, |z_n|^2)$

$\mu_A : \mathbb{C}^n \to R(A)^*_{\mathbb{R}}$

$\quad\;\; \cup \qquad\qquad\quad \cup$

$\quad \mu^{-1}(p) \qquad\quad p$

$\mu^{-1}(p) / T(A)$

$T(A) = R(A)_{\mathbb{R}} / R(A)$

$\Downarrow$

$(U(1))^n$

$\underline{H = \sum |z_i|^2}$

$X(p) = \mu^{-1}(p) / T(A)$

quasi-smooth

alg. quasi-proj

alg. variety

$$0 \to R(A) \to \mathbb{Z}^n \to M \to 0$$

$$U(1)^n$$

$$T = R(A)_{\mathbb{R}} / R(A) \cong U(1)^{n-d-1}$$

$$\mu^{-1}(p) \subset \mathbb{C}^n \to \mathbb{R}^n$$

$$(z_i) \mapsto |z_i|^2$$

Assume $\underline{p} \in \Sigma_{n-1} \longleftrightarrow$ vertex of $\mathrm{Sec}(A)$

$\updownarrow$

convex triang. of $\underline{\Delta = \mathrm{Conv}(A)}$

$\boxed{\text{convex triang. is unimodular.}}$

$p \in R(A)^{*}_{\mathbb{R}}$

$X(p)$ is smooth.

$$\underline{\mathrm{rk}\, H^i(X(p), \mathbb{Z})} = \begin{cases} 0 & i = 2k+1 \\ \Psi_k(A) & i = 2k. \end{cases}$$

$PH^{d-1}(T_f)$

$\cong$

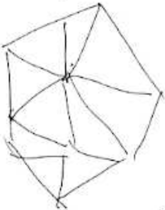
S_f^+

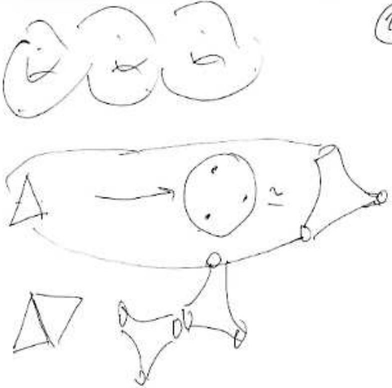
$R(A)_{\mathbb{R}}$

$H^k(X(p))$.

$R(A)_{\mathbb{R}}$

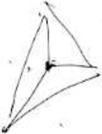
$d=2$

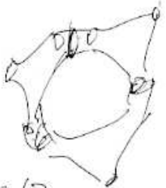




$Z_f \subseteq (\mathbb{C}^*)^2$

affine curve





$\ell^*(\Delta) = g(Z_f)$ $\quad g \quad PH^q(Z_f)$

$\dim H_1(\overline{Z}_f) = 2g$ $\quad \cup$

$\dim H = \ell^*(\Delta)$

$\psi(S^+_f) = (\ell(\Delta) - 3)t + \ell^*(\Delta)t^2$

$\psi(1) = 1 + \psi_1(\Delta)t + \ldots$

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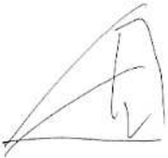
$H^i(X(p))$

$\dim H^2(X(p)) = \ell(\Delta) - 3$

$\dim H^4(X(p)) = \ell^*(\Delta)$



$\tilde{K}^0(X(p)) = K_0(X(p)) \oplus \mathbb{Z}$

$PH^1(Z_f) = C^+_f$