415A/515A Exam 1
Practice Problems

The following problems are related to the course material found in Sections 0-9 in the textbook. These problems do not reflect the types (or difficulty) of problems you may see on Exam 1 (in fact, these problems were selected having no knowledge of Exam 1 at all), nor does it cover every aspect of the material from those sections; however, they should all be problems you can complete with your knowledge of the material thus far. Please let me know if there are any errors or typos, and if anything could use clarification please don't hesitate to ask. Here is my email once again for reference: `rwilliams@math.arizona.edu`.

1. Let $S = \{w, x, y, z\}$ and $T = \{1, 2, 3, 4\}$, and define $f : S \to T$ and $g : S \to T$ by $f(w) = 2$, $f(x) = 4$, $f(y) = 1$, $f(z) = 2$ and $g(w) = 4$, $g(x) = 2$, $g(y) = 3$, $g(z) = 1$.
   (a) Is f injective? Is g injective? Is f surjective? Is g surjective?
   (b) Let $A = \{w, y\}$ and $B = \{x, y, z\}$. Determine each of the following subsets of T: $f(A)$, $g(B)$, $f(A \cap B)$, $g(A \cup B)$.
2. For each $n \in \mathbb{Z}$ let the mapping $f_n : \mathbb{Z} \to \mathbb{Z}$ be defined by $f_n(x) = nx$. For which values of n is f_n surjective? Injective?
3. Prove that if $f : S \to T$ and A and B are subsets of S, then $f(A \cup B) = f(A) \cup f(B)$.
4. Find all the solutions to the equation $z^3 + 2i = 0$.
5. Complete the following table in such a way that $*$ is commutative, has an identity element, and each element has an inverse:

$*$	w	x	y	z
w	y			x
x	z	w		
y				
z				w

6. Verify that $\{2^m 3^n : m, n \in \mathbb{Z}\} \subset \mathbb{R}$ is a group with respect to multiplication. Is it also a group with respect to addition? Why?
7. Find a subgroup of $\mathbb{Q}$ (under the operation $+$) that contains $\mathbb{Z}$ but is different from both $\mathbb{Z}$ and $\mathbb{Q}$.
8. Write 2 as a linear combination of 56 and 126.
9. Prove that if n is odd, then $\phi(2n) = \phi(n)$.
10. Prove that if $(a, b) = 1$, $a|m$ and $b|m$, then $ab|m$.
11. Let G be a group such that the order of each nonidentity element is 2. Prove that G is abelian.
12. Fill in the blanks in the following table to obtain a group isomorphic to $\mathbb{Z}/4\mathbb{Z}$:

$*$	a	b	c	d
a				
b				
c				
d				

13. Suppose that G and H are groups with $G \cong H$. Prove that if H has an element of order n, then H has an element of order n.