1. (Det. Computation) – Use the method of cofactor expansion along the *first column* to compute the determinant of

$$\begin{pmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ -2 & 2 & 1 \end{pmatrix}.$$

2. (Prop. of Det.) Let A and B be 5×5 matrices with rows a_1, a_2, a_3, a_4, a_5 and b_1, b_2, b_3, b_4, b_5, respectively. Suppose that $\det A = 3$ and the rows of B are given by: $b_1 = 2a_2$, $b_2 = a_5$, $b_3 = 5a_3 - 4a_1$, $b_4 = a_1$ and $b_5 = a_4$. Find $\det(B^T)^{-1}$.

3. (Prop. of Det.) Prove the following is true, or provide a counterexample showing it is false:

$$\det(A + B) = \det(A) + \det(B) \text{ for all } n \times n \text{ matrices } A, B.$$

4. (Prop. of Det.) Let A, B, C be $n \times n$ matrices such that $\det A = 2, \det B = -3$ and $\det C = 5$. Compute $\det(B^{-1}A^T AC^2)$.

5. (Eigenspace) The matrix $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ has eigenvalues 5 and -1. Find the eigenspace associated to each of these eigenvalues.

6. (Eigenvalue/Eigenvector) Let A be a 3×3 matrix with columns $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$. Suppose that the columns of A are related by the linear dependence relation $\mathbf{a}_1 - 2\mathbf{a}_2 + 5\mathbf{a}_3 = \mathbf{0}$. Use this information to find an eigenvector for A and find its corresponding eigenvalue.

7. (Char. Poly. / Eigenvalues) Compute the characteristic polynomial of the following matrix and use it to determine the eigenvalues:

$$\begin{pmatrix} 2 & 1 \\ -4 & 6 \end{pmatrix}.$$

8. (Char. Poly.) Suppose the characteristic polynomial of a square matrix A is given by $\lambda^4 - 5\lambda^3 + 6\lambda^2$. What is the size of the matrix A?

9. (Diagonalization - Possible) Diagonalize the following matrix if possible

$$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}.$$

10. (Diagonalization - Possible) Diagonalize the following matrix if possible

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 2 \\ -2 & 0 & 3 \end{pmatrix}.$$

11. (Diagonalization - Impossible) Diagonalize the following matrix if possible

$$A = \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}.$$

12. (Eigenvalues/Diagonalization) The matrix $A = \begin{pmatrix} 16 & -35 \\ 6 & -13 \end{pmatrix}$ has eigenvectors $\mathbf{v}_1 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$.

    (a) Determine the corresponding eigenvalues for $\mathbf{v}_1$ and $\mathbf{v}_2$.

    (b) Decompose the matrix A as PDP^{-1} where D is a diagonal matrix.

(c) Compute A^3 using your answer from (b).

13. (Orthogonality) Show the vectors $\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ -7 \end{pmatrix}$ are orthogonal.

14. (Orthogonality) Determine a nonzero vector that is orthogonal to the vector $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

15. (Orthogonality) Determine a nonzero vector that is orthogonal to both $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$.

16. (Properties of Dot Product / Norm) Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors in a given vector space and let c and d be nonzero scalars. Determine whether the following expressions are: a *scalar*, a *vector*, or if it *makes no sense*.

   (a) $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$
   (b) $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$
   (c) $\mathbf{u} \cdot \mathbf{v} + c\mathbf{w}$
   (d) $\mathbf{u} \cdot \mathbf{v} + c$
   (e) $c(\mathbf{u} \cdot \mathbf{v}) + d\mathbf{w}$
   (f) $\|\mathbf{u} + c\mathbf{v}\| + d$
   (g) $c\mathbf{u} \cdot d\mathbf{v} + \|\mathbf{w}\|\mathbf{v}$
   (h) $\|\mathbf{u} \cdot \mathbf{v}\|$

17. (Distance) Find the distance between the vectors $\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$.

18. (Orthonormal Basis) The vectors

$$\mathbf{u}_1 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} \frac{3}{5} \\ 0 \\ -\frac{4}{5} \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} \frac{4}{5} \\ 0 \\ \frac{3}{5} \end{pmatrix}$$

form an orthonormal basis for $\mathbb{R}^3$. Write the vector $\mathbf{v} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ as a linear combination of these basis vectors.

19. (Projection) Determine the orthogonal projection of the vector $\mathbf{v}$ onto the vector $\mathbf{u}$ given

$$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}.$$

20. (Gram-Schmidt) The following vectors form a basis for $\mathbb{R}^3$.

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}.$$

Use these vectors in the Gram-Schmidt process to construct an orthonormal basis for R^3.

21. (T/F - Det.) Let A be an $n \times n$ matrix and c a nonzero scalar, then $\det(cA) = |c| \det(A)$.

22. (T/F - Det.) Let A be an $n \times n$ matrix and c a nonzero scalar, then $\det(cA) = c\det(A)$.
23. (T/F - Det.) Let A be an $n \times n$ matrix and c a nonzero scalar, then $\det(cA) = c^n \det(A)$.
24. (T/F - Eigenvalue) If one of the eigenvalues of a matrix is zero, then the matrix is not invertible.
25. (T/F - Eigenvalue) The entries on the main diagonal of a matrix equal the eigenvalues of the matrix.
26. (T/F - Eigenvalue) The multiplicity of an eigenvalue is the number of eigenvectors which correspond to it.
27. (T/F - Eigenvalue) An eigenvector cannot correspond to more than one eigenvalue.
28. (T/F - Diagonalization) If A can be diagonalized, then A is invertible.
29. (T/F - Diagonalization) If A is invertible, then A can be diagonalized.