# Exam 3 for Math 215 Name: _____

**1. (12) Find the eigenvalues of the following matrices.**

**(a)** $A = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$.

**(b)** $B = \begin{bmatrix} 7 & 1 & 8 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{bmatrix}$.

**2. (15) The matrix** $A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix}$ **has eigenvalue -1.**

**(a)** $det(A + I) =$

**(b) Find all of the eigenvectors of A corresponding to the eigenvalue -1.**

**(c) Find a basis for the eigenspace of A corresponding to the eigenvalue -1.**

**3. (18)** $\begin{bmatrix} 5 & 6 & 6 \\ -6 & -7 & -6 \\ 6 & 6 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 & 0 \\ -0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

**The above equation represents the product:** $A = P \cdot D \cdot P^{-1}$

**(a) Find the eigenvalues of A.**

**(b) Find the determinant of A.**

**(c) How many eigenspaces are there?**

**(d) Find bases for the eigenspaces.**

**4. (12) Let $\vec{v} = [2, 1, 3, 4]^T$ and let L be the line determined by $\vec{v}$.**

**(a) Find the projection of $\vec{u} = [1, 8, 7, 2]^T$ onto the line L.**

**(b) Find the distance from the point $(1, 8, 7, 2)$ to the line L.**

**5. (10) Let $W = \text{span}\{[1, 7, 2, 3]^T, [3, -2, 7, 1]^T\}$. Find the projection of $\vec{u} = [4, 2, 8, 7]^T$ onto $W$.**

**6. (8) Let $\vec{u} = [1, 8, 7, 2, 7]^T$. Let $W = \{\vec{w} \in R^5 : \vec{u} \cdot \vec{w} = 0\}$. Determine the dimension of W.**

**7. (12) $A$ is a 4x4 matrix with given characteristic polynomial. What conditions, if any, must be imposed so that $A$ is diagonalizable?**

**(a)** $(\lambda - 2)^2 \cdot (\lambda - 4) \cdot (\lambda + 7)$.

**(b)** $(\lambda) \cdot (\lambda - 2) \cdot (\lambda - 4) \cdot (\lambda + 7)$.

**(10) Label each of the following statements as True (T) or False (F).**

**(a) $A$ has an eigenvalue of $0$ if and only if $det(A) = 0$. T or F**

**(b) Let $W$ be a subspace of dimension 3 in $R^7$. Then $W$ has an orthonormal basis. T or F**

**(c) Let $A$ be an n x n matrix whose columns are orthonormal. The $det(A) = 1$. T or F**

**(d) If $\vec{u}$ is an eigenvector of $A$, then $\vec{u}$ is an eigenvector of $A^2$. T or F**

**(e) If $U$ is a matrix with non-zero orthogonal columns, then $U^T \cdot U$ is a diagonal matrix. T or F**