Exam 2 for Math 313 Name: _____

1.

2. (11) Let $A = \begin{bmatrix} 3 & 8 \\ 4 & 7 \end{bmatrix}$.

   a) Are the columns of A linearly independent? Explain.

   b) If A has an inverse, find it.

   c) $A \cdot A^{-1} =$

3. (10) Let $A = \begin{bmatrix} -2 & 5 \\ 8 & -20 \end{bmatrix}$. Find all 2 x 2 matrices B so that $A \cdot B = 0$.

4. (14) Let $A = \begin{bmatrix} -3 & 52 \\ 7 & -18 \end{bmatrix}$. Determine if the following two subsets of M_{2x2} are subspaces.

   (a) Let H be the set of all 2 x 2 matrices B so that $A \cdot B = B \cdot A$.

   (b) Let H be the set of all 2 x 2 matrices B so that $A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

5. (12) Find the inverses of the following matrices.

   (a) $A_1 = \begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}$

   (b) $A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   (c) $A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

6. (8) Let A be a 9 x 13 matrix. If a basis for Nul(A) consists of five vectors, can the columns of A span R^9? Explain.

7. (33) Let $A = \begin{bmatrix} 57 & -44 & -67 & -44 & -93 & -8 \\ -59 & 53.5 & 133 & 63 & 182 & 12 \\ -9 & 4.5 & -9 & 1 & -14 & 0 \\ -10 & 7 & 6 & 6 & 8 & 1 \\ -28 & 22 & 36 & 22 & 48 & 4 \end{bmatrix}$ and $A_1 = rref(A) = \begin{bmatrix} 1 & 0 & 5 & 0 & 3 & 0 \\ 0 & 1 & 8 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) A can be thought of as function from R^n to R^m. What are n and m?

(b) Find a basis for Nul(A).

(c) Suppose that $A \cdot X = A \cdot Y$. Show that $X - Y \in$ Nul(A).

(d) Find a basis for the Col(A).

(e) If the columns of A are not linearly independent, express one of the columns of A as a linear combination of the others.

(f) Let X and B be the column vectors, $X = [-1, 1, 2, 1, -3, 2]^T$ and $B = [-16, -80.5, 46.5, 13, 8]^T$, and that $A \cdot X = B$. Find all Y so that $A \cdot Y = B$.

8. (12) Label each of the following statements as True (T) or False (F).

(a) If A and B have the same rref, then $A = B$. T or F

(b) If A and B have the same inverse, then $A = B$. T or F

(c) If the $rref(A)$ has a pivot element in every row, then the columns of A are linearly independent. T or F

(d) $\{1, x, x^2, x^3, x^4\}$ is a basis for the vector space of polynomials of degree 4 or less. T or F

(e) Every n x n matrix A is the product of n x n elementary matrices. T or F

(f) If A and B are invertible n x n matrices, then $A + B$ is invertible. T or F