Exam 1 for Math 215, Name: _____

1. [20] Find the RREF for each of the following matrices.

$$\mathrm{A}_1 = \begin{bmatrix} 2 & 6 & 2 & 14 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 3 \\ 3 & 8 & 7 & -12 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -.7 & 0 & 8 & -1 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 10 \\ 4 \\ 5 \\ -7 \\ 2.1 \\ 47 \end{bmatrix}$$

2. [6] Does there exist a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^3$ that is onto? Explain.

3. [6] Let A be a 5x7 matrix. Are the columns of A linearly independent? Explain.

4. Let B be a matrix whose RREF is $B_1 = \begin{bmatrix} 1 & 0 & -8 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) [4] B has _____ rows and _____ columns.

   (b) [6] Are the columns of B linearly independent? Explain.

   (c) [8] Is it possible to write one of the columns of B as a linear combination of the other columns? If so, write such a linear combination.

The items on this page are a continuation of problem 3 and $\mathrm{B}_1 = \begin{bmatrix} 1 & 0 & -8 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(d) [10] Solve the system of equations, $B \cdot X = 0$ and write your solution in vector form.

(e) [6] Describe the solution to the previous item geometrically.

(f) [8] View B as an augmented matrix. Describe the solutions, if any, of the system of equations represented by this augmented matrix.

5. [8] Let $\{v_1, v_2, ..., v_k\}$ be a set of k vectors in $\mathbf{R}^n$. If v_1 is the zero vector then determine whether or not the set of k vectors is linearly independent.

6. Let $T(x, y, z) = (4x + 7y + d, -y + z + d^2, x + y, -10z)$

   [5] a) For what values of d is T a linear transformation? Explain.

   [10] b) For those values of d for which T is a linear transformation, write down the corresponding matrix that represents the linear transformation.