

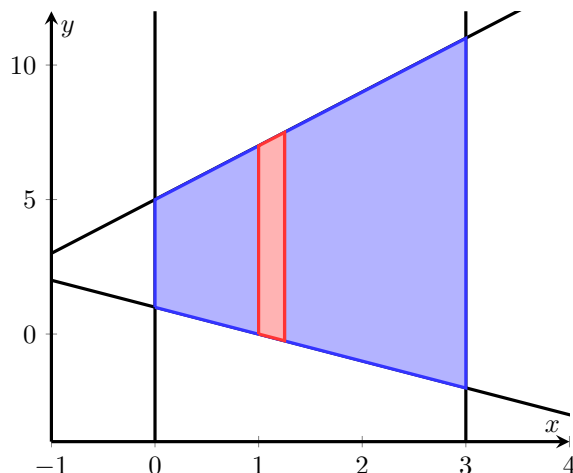
Homework 5 Solutions

Sections 8.2

1. The base of a solid is the region in the xy -plane bounded by the y -axis and the lines $y = 1 - x$, $y = 2x + 5$ and $x = 3$. Each cross section perpendicular to the x -axis is an equilateral triangle. Find the volume of the solid.

Solution.

Graphing the given equations we see the base of our solid will be given by:



We slice this region into vertical strips of width Δx and compute the volume of the object having this strip as a base.

To compute the volume of this slice we first need to compute the area of the vertical equilateral triangle sticking up from this slice, then simply multiply by the width, Δx .

The length of the base (and hence of all of the sides) at x of the equilateral triangle will be given by $(2x + 5) - (1 - x) = 3x + 4$. Now, recall that the area of an equilateral triangle is given by

$$\text{Area} = \frac{\sqrt{3}}{4}(\text{Length of Side})^2.$$

(Try deriving this formula – it's not too hard!) So the area of our triangle is:

$$\text{Area} = \frac{\sqrt{3}}{4}(3x + 4)^2,$$

and hence the volume of our slice is given by

$$\text{Volume of Slice} = \frac{\sqrt{3}}{4}(3x + 4)^2 \Delta x.$$

Thus, to compute the total volume of the object we just need to integrate:

$$\int_0^3 \frac{\sqrt{3}}{4}(3x + 4)^2 dx = \frac{237\sqrt{3}}{4} \approx 102.6.$$