Writing Guidelines — Calculus II — Spring 2015

The following is an in-progress list of guidelines for written work. It is by no means an exhaustive list. You should expect all of these rules to contribute to your grade on all written work in this class.

Layout

(L1) Multiple pages must be **stapled together**. Crazy corner origami is *not* a staple.

(L2) There should be **no fringes** on your paper.

(L3) Arrange your work in **one column**. There is no second column, there is only a second page. Your attempt to "save paper" makes both of our lives more difficult.

(L4) Leave **space between problems**. On lined paper, leave at least one empty line between problems.

General Notation

(N1) Make use of **proper notation** and **appropriate spacing** to make your work clear and easy to follow.

Example: Suppose $f(2) = 4$, $f'(2) = 2.1$, and $k(x) = (f(x))^{-1}$. Find $k'(2)$.

- BAD:

$$\frac{d}{dx} f^{-1} \cdot \frac{d}{dx} f(x)$$

$$-\frac{1}{f(x)^2} \cdot f'(x) \quad -\frac{1}{16}^{\cdot 2.1} \quad \boxed{\frac{-2.1}{16}}$$

- GOOD:

$$\begin{aligned} k(x) &= (f(x))^{-1} \\ k'(x) &= -1(f(x))^{-2} \cdot f'(x) \\ k'(2) &= \frac{-1}{(f(2))^2} \cdot f'(2) \\ &= \frac{-1}{4^2} \cdot 2.1 \\ &= \boxed{\frac{-2.1}{16}} \end{aligned}$$

(N2) Use the **equal sign** properly. The symbol "=" means that the expression on the left and the expression on the right are algebraically the same.

(a) When you say that two expressions are equal, they *must actually be equal.*

Example: Suppose $f(x) = e^{5x}$. Find $f'(2)$.

- BAD:

$$5 \cdot e^{5x} = 5 \cdot e^{5\cdot 2} = 5e^{10}$$

- GOOD:

$$f'(x) = 5e^{5x}$$
$$f'(2) = 5e^{5\cdot 2} = \boxed{5e^{10}}$$

Example: Find $\cos^{-1}(-1)$.

- BAD:

$$\cos^{-1}(-1) = \pi = 3.14$$

π and 3.14 are not equal to each other.

- ALSO BAD:

$$\cos^{-1}(-1) = \pi = 3.14159$$

Again, π is not equal to 3.14159.

- GOOD:

$$\cos^{-1}(-1) = \pi$$

Note: $\pi \approx 3.14$, or $\pi \approx 3.14159$ is the correct way to write what you meant in the previous example. But you **shouldn't do this either** - see Rule (N7).

(b) The equals sign does **not** mean "the next step is". The symbols $\to$ and $\Rightarrow$ do **not** mean "the next step is" either.

Example: Solve the equation $e^{x^2} = 2$.

- COMPLETE NONSENSE:

$$e^{x^2} = 2 = \ln(e^{x^2}) = \ln 2 = x^2 = \ln 2 = x = \pm \ln 2$$

- ALSO NONSENSE:

$$e^{x^2} = 2 \to \ln(e^{x^2}) = \ln 2 \to x^2 = \ln 2 \to x = \pm \ln 2$$

The symbol '$\to$' in this class means "approaches", and is used in the context of limits. What is written above is "the equation $e^{x^2} = 2$ *approaches* the equation $\ln(e^{x^2}) = \ln 2$". This doesn't make any sense.

- BAD:

$$e^{x^2} = 2 \Rightarrow \ln(e^{x^2}) = \ln 2 \Rightarrow x^2 = \ln 2 \Rightarrow x = \pm \ln 2$$

The symbol '$\Rightarrow$' represents an *implication.* The example given is not strictly incorrect, but the work is difficult to follow. The same work can be written **more clearly without** this symbol (see the GOOD work below).

If you move on to take a class like Linear Algebra, or Sets and Proofs, then you will learn how to use this symbol correctly. For now, **avoid the symbol '$\Rightarrow$' completely** in your writing.

- GOOD:

$$\begin{aligned} e^{x^2} &= 2 \\ \ln(e^{x^2}) &= \ln 2 \\ x^2 &= \ln 2 \\ x &= \boxed{\pm \ln 2} \end{aligned}$$

Vertically aligned math is better than horizontally aligned math.

(c) Do not use $\to$ or $\Rightarrow$ when you mean $=$.

Example: Suppose $f(4) = 8$ and $f'(4) = 3$. Find $(f^{-1})'(8)$.

- BAD:

$$(f^{-1})'(8) \to \frac{1}{f'(f^{-1}(8))} \to \frac{1}{f'(4)} \to \frac{1}{3}$$

- ALSO BAD:

$$(f^{-1})'(8) \Rightarrow \frac{1}{f'(f^{-1}(8))} \Rightarrow \frac{1}{f'(4)} \Rightarrow \frac{1}{3}$$

- GOOD:

$$(f^{-1})'(8) = \frac{1}{f'(f^{-1}(8))} = \frac{1}{f'(4)} = \frac{1}{3}$$

(N3) **Label** your expressions.

Example: Find the first and second derivatives of $f(x) = 2x^4 + 3x^2 - 5x + 2$.

- BAD:

$$\begin{aligned} &2x^4 + 3x^2 - 5x + 2 \\ &8x^3 + 6x - 5 \\ &24x^2 + 6 \end{aligned}$$

- GOOD:

$$\begin{aligned} f(x) &= 2x^4 + 3x^2 - 5x + 2 \\ f'(x) &= 8x^3 + 6x - 5 \\ f''(x) &= 24x^2 + 6 \end{aligned}$$

(N4) Each step should follow clearly from the previous one. **All relevant work** must be shown, but **don't include scratch work**.

Example: Find the indefinite integral of $f(t) = 1 + (\cos t + \sin t)^2$.

- BAD:

$$\int 1 + (\cos t + \sin t)^2 \, dt$$

$$\cos^2 t + 2\cos t \sin t + \sin^2 t$$

$$1 + 2\cos t \sin t$$

$$1 + \sin(2t)$$

$$= \int 1 + 1 + \sin(2t) \, dt$$

$$= 2t - \frac{1}{2}\cos(2t) + C$$

(Scratch work does not belong in a final document.)

- ALSO BAD:

$$\begin{aligned} \int 1 + (\cos t + \sin t)^2 \, dt &= \int 2 + \sin(2t) \, dt \\ &= 2t - \frac{1}{2}\cos(2t) + C \end{aligned}$$

(Relevant work is missing - how did $1 + (\cos t + \sin t)^2$ turn into $2 + \sin(2t)$?)

- GOOD:

$$\begin{aligned} \int 1 + (\cos t + \sin t)^2 \, dt &= \int 1 + \cos^2 t + 2\cos t \sin t + \sin^2 t \, dt \\ &= \int 1 + 1 + 2\cos t \sin t \, dt \\ &= \int 2 + \sin(2t) \, dt \\ &= \boxed{2t - \frac{1}{2}\cos(2t) + C} \end{aligned}$$

(N5) Use **variable names as given**.

Example: Find the derivative of $p = q(\ln q)$.

- BAD: Let $q = x$ and $p = f(x)$.

$$\begin{aligned} f'(x) &= 1 \cdot \ln x + x \cdot \frac{1}{x} \\ &= \boxed{\ln x + 1} \end{aligned}$$

- GOOD:

$$\begin{aligned} \frac{dp}{dq} &= 1 \cdot \ln q + q \cdot \frac{1}{x} \\ &= \boxed{\ln q + 1} \end{aligned}$$

(N6) **Avoid** using the word '`it`'.

- BAD:

  Since $f'(4)$ is positive, it is increasing.

  What's increasing? The derivative? The value $f'(4)$?

- GOOD:

  Since $f'(4)$ is positive, $f(x)$ is increasing at $x = 4$.

(N7) Give **exact answers**. There are *only two* exceptions: 1) If you are specifically asked to approximate, or 2) If you are solving a real-world application type problem. An "exact answer" is one in which you have **not approximated anything**.

Example: Solve the equation $\theta^2 - 40 = 0$.

- BAD:

$$\begin{aligned} \theta^2 &= 40 \\ \theta &= \pm\sqrt{40} \\ &\approx \boxed{\pm 6.32} \end{aligned}$$

- GOOD:

$$\begin{aligned} \theta^2 &= 40 \\ \theta &= \boxed{\pm\sqrt{40}} \end{aligned}$$

Exception: Saying "\$6.32" is generally better than saying "$\sqrt{40}$ dollars".

(N8) **Clearly mark your answers**. Box them, or circle them, or write them in a provided answer blank. In addition, **don't put down multiple answers**, or multiple versions of work.

Example: Find the derivative of $f(x) = \frac{x^3}{3} - 4x + e$.

- BAD:

$$\begin{aligned} f'(x) &= 3 \cdot \frac{x^2}{3} - 4 + 0 \\ &= x^2 - 4, \text{ OR } (x-2)(x+2) \end{aligned}$$

- GOOD:

$$\begin{aligned} f'(x) &= 3 \cdot \frac{x^3}{3} - 4 + 0 \\ &= x^2 - 4 \\ &= \boxed{(x-2)(x+2)} \end{aligned}$$

(N9) Evaluate **trig** and **inverse trig** functions whenever possible using the unit circle. Keep in mind Rule (N7).

Example:

- BAD: $\boxed{\tan^{-1}(1)}$ and $\cos(1) \approx \boxed{.54}$.
- GOOD: $\tan^{-1}(1) = \boxed{\frac{\pi}{4}}$ and $\boxed{\cos(1)}$.

(N10) **The more words you use, the better.** In particular, **do not use** any of the following symbols:

(a) Don't use $\therefore$ Instead, use the word "then" or "therefore".

(b) Don't use $\because$ Instead, use the word "because" or "since".

(c) Don't use $\ni$

(d) Don't use $\Rightarrow$

Calculus Notation

The following rules are specifically about the notation used in calculus.

(C1) **Limit notation**.

(a) The notation $\lim_{x \to a} f(x)$ must include *all three* parts: $\lim, x \to a$, and $f(x)$. If you omit any one of those three parts in your limit notation, it becomes nonsense.

Example: Find $\lim_{x \to 2} f(x)$, where $f(x) = x^2 + 3$

- NONSENSE:

$$\lim = 2^2 + 3 = 7$$

- ALSO NONSENSE:

$$\lim_{x \to 2} = 2^2 + 3 = 7$$

- *ALSO* NONSENSE:

$$\lim f(x) = 2^2 + 3 = 7$$

- GOOD:

$$\begin{aligned} \lim_{x \to 2} f(x) &= \lim_{x \to 2} x^2 + 3 \\ &= 2^2 + 3 \\ &= \boxed{7} \end{aligned}$$

(b) When calculating limits, you must pay careful attention to whether your equal signs are used properly (see Rule (N2)). In general, $f(x)$ is an *algebraic expression*, while $\lim_{x\to a} f(x)$ is a *number*.

Example: Calculate $\lim_{s\to 3} \frac{s^2-9}{s-3}$.

- BAD:

$$\begin{aligned}\lim_{s\to 3} \frac{s^2-9}{s-3} &= \frac{(s+3)\cancel{(s-3)}}{\cancel{s-3}} && (1)\\ &= s+3 \\ &= 3+3 && (2)\\ &= 6\end{aligned}$$

  The equalities claimed in steps (1) and (2) are just plain wrong!

- GOOD:

$$\begin{aligned}\lim_{s\to 3} \frac{s^2-9}{s-3} &= \lim_{s\to 3} \frac{(s+3)\cancel{(s-3)}}{\cancel{s-3}} \\ &= \lim_{s\to 3} s+3 \\ &= 3+3 \\ &= 6\end{aligned}$$

(C2) **Derivative notation**. There are many different types of notation that can be used for derivatives. Each of them is used in slightly different situations. In this class, we will mainly use two different types of notation: *prime notation* (e.g. $f'(x), h''(y)$), and *Leibniz notation* (e.g. $\frac{dy}{dx}, \frac{d^2z}{dt^2}$).

(a) **Use consistent notation**. Generally, either prime notation or Leibniz notation will be appropriate - but not both. Choose your derivative notation appropriately.

Example: Find the derivative of $y = \cosh x + 2x$.

- BAD: $f'(x) = \sinh x + 2$. *(What's f? There's no 'f' in the problem statement...)*
- GOOD: $\frac{dy}{dx} = \sinh x + 2$.

Example: Find the derivative of $f(x) = 3^x$.

- BAD: $\frac{dy}{dx} = \ln 3 \cdot 3^x$. *(What's y? There's no 'y' in the problem statement...)*
- GOOD: $f'(x) = \ln 3 \cdot 3^x$.

(b) Use Leibniz notation correctly. **Say what you mean**.

- $\frac{dy}{dx}$ means the derivative of y with respect to x,
- $\frac{d}{dx}(\log x + \cosh x)$ means the derivative of $\log x + \cosh x$ with respect to x,
- $\frac{dy}{dx}(\log x + \cosh x)$ means take the derivative of y with respect to x, then multiply that expression by $(\log x + \cosh x)$.

Example: Find $\frac{dy}{dx}$ if $y^2 + \ln y = 5x$.

- BAD:

$$\begin{aligned} \frac{dy}{dx}(y^2) + \frac{dy}{dx}(\ln y) &= \frac{dy}{dx}(5x) \\ 2y\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} &= 5 \\ &\vdots \end{aligned}$$

- GOOD:

$$\begin{aligned} \frac{d}{dx}(y^2) + \frac{d}{dx}(\ln y) &= \frac{d}{dx}(5x) \\ 2y\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} &= 5 \\ &\vdots \end{aligned}$$

(c) Leibniz notation requires both the "operator part" ($\frac{d}{dx}$, $\frac{d^2}{dt^2}$) and the function you are differentiating (y, $\log x + \cosh x$, $-32t^2 + 60$).

Example: Find the derivative of the function $y = \tan z$.

- BAD:

$$\frac{d}{dz} = \sec^2 z$$

  The left hand side is missing a function - what am I differentiating?

- BAD:

$$\frac{dy}{dz}(\tan z) = \sec^2 z$$

  The left hand side here says: take the derivative of y with respect to z, then multiply that by $\tan z$. This is not what we wanted to do.

- GOOD:

$$\frac{dy}{dz} = \frac{d}{dz}(\tan z) = \sec^2 z$$

(d) When evaluating derivatives at a point, pay careful attention to whether your equal signs are used properly (see Rule (N2)). In particular, $f'(x)$ and $\frac{dy}{dx}$ are *algebraic expressions*, while $f'(a)$ and $\frac{dy}{dx}\big|_{x=a}$ are *numbers*.

Example: Find the derivative of $y = \cos x$ at $x = \frac{\pi}{2}$.

- BAD:

$$\frac{dy}{dx} = -\sin\frac{\pi}{2} = -1$$

- GOOD:

$$\begin{aligned} \frac{dy}{dx} &= -\sin x \\ \frac{dy}{dx}\bigg|_{x=\frac{\pi}{2}} &= -\sin\frac{\pi}{2} \\ &= -1 \end{aligned}$$

(C3) **Integral notation**. When calculating definite integrals, you must pay careful attention to whether your equal signs are used properly. (See Rule (N2) - Rule (N2) is very important!)

Example: Find the area under the graph of $f(x) = x^2 + 1$ from $x = 1$ to $x = 2$.

- BAD:

$$\begin{aligned} \int_1^2 3x^2 + 1\,dx &= x^3 + x + C && (3) \\ &= (2^3 + 2) - (1^3 + 1) && (4) \\ &= 10 - 2 \\ &= 8 \end{aligned}$$

The equalities claimed in steps (3) and (4) are just plain wrong! In step (3), the left hand side is a *number*, and the right hand side is an *algebraic expression*. In step (4), the left hand side is an algebraic expression, and the right hand side is a number.

- GOOD:

$$\begin{aligned} \int_1^2 3x^2 + 1\,dx &= \left(x^3 + x\right)\Big|_1^2 \\ &= (2^3 + 2) - (1^3 + 1) \\ &= 10 - 2 \\ &= 8 \end{aligned}$$

Please **complete**, **detach**, and **submit this page** as Homework 0 Part 2.

"I have read and completely understand the Writing Guidelines for this class."

Name: ____

Signature: ____ Date: _____________