MATH 129
FINAL EXAM REVIEW PACKET ANSWERS
(Spring 2014)

1. $\int_0^3 \left(10\pi \sin\left(\frac{\pi}{3}t\right) + 30\right) dt = 150$ people

2. $\int_1^2 f(5-2x)dx = \frac{7}{2}$ Let $u = 5-2x$ and change the endpoints.

3. a) $\int \frac{t}{\sqrt{t+1}} dt = \frac{2}{3}(t+1)^{3/2} - 2(t+1)^{1/2} + c$ By the method of substitution with $u = t+1$.
You can also use integration by parts with $u = t$ and $v' = (t+1)^{-1/2}$. The result is equivalent, just written in a different form. $\int \frac{t}{\sqrt{t+1}} dt = 2t(t+1)^{1/2} - \frac{4}{3}(t+1)^{3/2} + c$

b) $\int \left(\frac{1}{z^2} + A\right)^2 dz = -\frac{1}{3z^3} - \frac{2A}{z} + A^2 z + c$ Distribute.

c) $\int 3^x e^x dx = \frac{1}{\ln(3e)}(3e)^x + c$ Rewrite $3^x e^x = (3e)^x$.

d) $\int_0^1 \frac{\arctan y}{1+y^2} dy = \frac{\pi^2}{32}$ Substitution and change endpoints. Always simplify answer.

4. a) $\int \frac{\ln\left(z^2+1\right)}{z^2} dz = -\frac{1}{z}\ln\left(z^2+1\right) + 2\arctan z + c$ Let $u = \ln(z^2+1)$ and $v' = \frac{1}{z^2}$.

b) $\int x \arcsin(x^2) dx = \frac{1}{2}x^2 \arcsin(x^2) + \frac{1}{2}\sqrt{1-x^4} + c$ First make a substitution with $w = x^2$.
Then let $u = \arcsin(w)$ and $v' = 1$.

c) $\int_0^1 x \cdot g''(x) dx = 3$ Let $u = x$ and $v' = g''$ for the first integration by parts.

5. a) $\int \cos^2(3\theta+2) d\theta = \frac{1}{6}\cos(3\theta+2)\sin(3\theta+2) + \frac{1}{6}(3\theta+2) + c$ Let $u = 3\theta+2$ before using table formula # 18. If you use another approach, your answer will look different.

b) $\int \frac{2}{4t^2-9}\,dt = \frac{1}{6}\left(\ln|2t-3| - \ln|2t+3|\right) + c$ Let $u = 2t$ and factor the denominator before using table formula # 26. If you use another approach, your answer will look different.

c) $\int \frac{dy}{\sqrt{y^2+8y+15}} = \ln\left|(y+4) + \sqrt{y^2+8y+15}\right| + c$ Complete the square before using table formula # 29.

d) $\int \frac{\sin(4\alpha)}{\cos^2(4\alpha) - \cos(4\alpha)}\,d\alpha = \frac{1}{4}\left(\ln|\cos(4\alpha)| - \ln|\cos(4\alpha) - 1|\right) + c$ Let $u = \cos(4\alpha)$ and factor the denominator before using table formula # 26.

6. a) $\int \frac{3y^3+5y-1}{y^3+y}\,dy = 3y - \ln|y| + \frac{1}{2}\ln|y^2+1| + 2\arctan(y) + c$ First do long division, then use partial fractions $\frac{A}{y} + \frac{By+C}{y^2+1}$.

b) $\int \frac{5z-28}{6z^2+z-40}\,dz = \frac{4}{3}\ln|3z+8| - \frac{1}{2}\ln|2z-5| + c$ Use partial fractions $\frac{A}{3z+8} + \frac{B}{2z-5}$.

c) $\int \frac{dx}{(5-x^2)^{3/2}} = \frac{1}{5}\frac{x}{\sqrt{5-x^2}} + c$ Let $x = \sqrt{5}\sin(\theta)$.

d) $\int \frac{dt}{t^2\sqrt{1+t^2}} = -\frac{\sqrt{1+t^2}}{t} + c$ Let $t = \tan(\theta)$.

7. a) $\int_1^3 f'(x)e^{f(x)}\,dx = e^{11} - e^7$ Let $u = f(x)$.

b) $\int_1^e \frac{f'(\ln x)}{x}\,dx = 2$ Let $u = \ln x$.

8. $v = k(R^2 - r^2)$. $\frac{1}{R-0}\int_0^R k(R^2 - r^2)\,dr = \frac{2}{3}kR^2$

9. $E(X) = \int_{-\infty}^{\infty} xf(x)\,dx = \int_{-\infty}^{0} xf(x)\,dx + \int_0^{\infty} xf(x)\,dx = 0 + \int_0^{\infty} x\frac{1}{7}e^{-x/7}\,dx = 7$

Use integration by parts or the table of integrals. Remember to use proper notation.

10. $\int_0^1 e^{-t^2} dt \approx \frac{1}{2}e^{-1/16} + \frac{1}{2}e^{-9/16} \approx 0.75459794$

11. a), d), and e)

12. Trap, Right, Mid, and Left

13. a) The integral converges. $\int_0^\infty \frac{1}{x^2+4} dx = \frac{\pi}{4}$ Use table formula # 24.

b) The integral converges. $\int_1^\infty \frac{1}{2^x} dx = \frac{1}{2\ln 2}$.

c) The integral diverges. $\int_0^1 \frac{e^x}{\left(e^x-1\right)^2} dx = \infty$ Let $u = e^x - 1$.

d) The integral converges. $\int_{\pi/6}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} dx = 2\sqrt{\frac{\sqrt{3}}{2}}$ Let $u = \cos x$

e) The integral diverges. $\int_1^\infty \frac{dx}{(x-2)^3} = \int_1^2 \frac{dx}{(x-2)^3} + \int_2^\infty \frac{dx}{(x-2)^3}$. The first integral diverges.

f) The integral converges. $\int_5^\infty \frac{du}{u^2-16} = -\frac{1}{8}\ln\left(\frac{1}{9}\right) = \frac{1}{8}\ln(9)$ Use table formula # 26.

14. $\int_m^\infty e^{-\left(\frac{x-m}{s}\right)^2} dx = \frac{s\sqrt{\pi}}{2}$ Let $u = \frac{x-m}{s}$ and change the endpoints.

15. a) The integral converges. Rewrite as $\int_0^\infty a \cdot f(x)dx = a\int_0^\infty f(x)dx$.

b) The integral converges. Let $u = ax$.

c) The integral diverges. Rewrite as $\int_0^\infty \left(a + f(x)\right)dx = \int_0^\infty adx + \int_0^\infty f(x)dx$.

d) The integral converges. Let $u = a + x$.

16. a) The integral converges. By comparison with $\int_2^{\infty} \frac{d\theta}{\theta^{3/2}}$.

b) The integral converges. By comparison with $\int_1^{\infty} \frac{1}{(x+3)^3} dx = \int_4^{\infty} \frac{1}{u^3} du$.

c) The integral diverges. By comparison with $\int_1^{\infty} \frac{1}{x} dx$

d) The integral diverges. $\lim_{x\to\infty} \frac{x^5}{e^{-x}+1} = \infty$. In order for the improper integral to converge, the integrand must approach 0.

17. b) and c)

18. a) volume of slice $\approx 16\sqrt{36-(6-x)^2}\Delta x$ Using Pythagorean Theorem.

b) volume of solid $\approx \sum 16\sqrt{36-(6-x_i)^2}\Delta x$ Using the notation of the text.

c) volume = $\int_0^6 16\sqrt{36-(6-x)^2} dx$

19. a) $\int_0^1 3\sqrt{x}dx + \int_1^2 (6-3x)dx$ b) $\int_0^3 \left(\frac{6-y}{3} - \frac{y^2}{9}\right) dy$

20. a) $75\pi - \int_0^3 \pi(5e^{-x})^2 dx = \frac{(125+25e^{-6})}{2}\pi$ b) $\int_0^3 \pi\left(5-5e^{-x}\right)^2 dx = \frac{(75+100e^{-3}-25e^{-6})\pi}{2}$

21. a) $\int_0^8 \pi\left(y^{1/3}\right)^2 dy = \frac{96}{5}\pi$ b) $\int_0^8 \pi\left(y^{1/3}+2\right)^2 dy - 32\pi = \frac{336}{5}\pi$

22. $\int_0^2 \left(\pi\left(f(x)\right)^2 - \pi\left(g(x)\right)^2\right) dx + \int_2^7 \left(\pi\left(g(x)\right)^2 - \pi\left(f(x)\right)^2\right) dx$

23. a) $\int_0^{\pi} (\sin x)^2 dx = \frac{\pi}{2}$ b) $\int_0^{\pi} \frac{1}{2}\pi\left(\frac{\sin x}{2}\right)^2 dx = \frac{\pi^2}{16}$

24. Left hand rule:

$$10\cdot\pi\left(\frac{26}{2\pi}\right)^2+10\cdot\pi\left(\frac{22}{2\pi}\right)^2+10\cdot\pi\left(\frac{18}{2\pi}\right)^2+10\cdot\pi\left(\frac{12}{2\pi}\right)^2+10\cdot\pi\left(\frac{6}{2\pi}\right)^2=\frac{4160}{\pi}\text{ cubic inches}$$

Right hand rule:

$$10\cdot\pi\left(\frac{22}{2\pi}\right)^2+10\cdot\pi\left(\frac{18}{2\pi}\right)^2+10\cdot\pi\left(\frac{12}{2\pi}\right)^2+10\cdot\pi\left(\frac{6}{2\pi}\right)^2+10\cdot\pi\left(\frac{2}{2\pi}\right)^2=\frac{2480}{\pi}\text{ cubic inches}$$

Trapezoid rule: $\frac{3320}{\pi}$ cubic inches Use the average of the left and right hand rules.

25. a) $\int_0^1\left(\pi\left(y^3\right)^2-\pi\left(\sqrt{y}\right)^2\right)dy$ b) $\int_0^\infty \pi\left(\frac{1}{x^2+1}\right)^2 dx$

26. $\int_0^5 \left(2+0.5\cosh x\right)dx=10+0.5\sinh 5$ grams

27. a) $\int_0^8 \delta(x)\cdot 2\pi x dx$ b) $\int_{-8}^8 \delta(x)\cdot 2\sqrt{64-x^2}dx$

28. $\int_0^{25}\left(-\frac{8}{5}h+90\right)\pi(10)^2 dh=175,000\pi$ pounds

29. a) $a_n=\frac{(-1)^n 2n}{(n+2)^2}$ b) $\lim_{n\to\infty} a_n=-\frac{3}{5}$ $\lim_{n\to\infty} b_n=5$

30. a) $\frac{(3/64)\left(1-(1/4)^8\right)}{1-1/4}=\frac{65535}{1048576}$ b) $\frac{3/4}{1-1/4}=1$

31. $P_n=\frac{(0.05)(200)\left(1-0.05^{n-1}\right)}{1-0.05}$ $Q_n=\frac{(200)\left(1-0.05^n\right)}{1-0.05}$ $n=1,2,3,\cdots$

32. a) The series converges. $\int_2^\infty \frac{1}{x(\ln x)^2}dx = \frac{1}{\ln 2}$ Use the method of section 7.7 to evaluate the improper integral.

b) The series diverges. $\int_1^\infty \frac{3x^2+2x}{\sqrt{x^3+x^2+1}}dx = \infty$ Use the method of section 7.7 to evaluate the improper integral.

33. a) The series diverges. $\lim_{n\to\infty} \frac{e(n)^2}{2(n+1)^2} = \frac{e}{2} > 1$

b) The series converges. $\lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{1}{4} < 1$

34. The Ratio Test gives us 1 and does not tell us anything about the convergence or divergence of the series.

35. The given series is conditionally convergent. $\sum_{k=5}^{\infty} \frac{(-1)^{k-1}}{k(\ln k)}$ converges by the Alternating Series Test, while $\sum_{k=5}^{\infty} \left| \frac{(-1)^{k-1}}{k(\ln k)} \right|$ diverges by the Integral Test.

36. a) and b)

37. a) False b) True c) False d) True e) False

38. a) The radius of convergence is $R = 3$. The interval of convergence is $(-7,-1)$.
b) The radius of convergence is $R = \infty$. The interval of convergence is $(-\infty,\infty)$.
c) The radius of convergence is $R = 0$. The series only converges for $x = 1$.

39. a) True b) True c) Impossible to determine.

40. $P_2(x)=4+\frac{1}{3}(x-1)-\frac{1}{144}(x-1)^2 \qquad f(2)\approx P_2(2)=\frac{623}{144}\approx 4.3264$

41. The sign of c_0 cannot be determined, $c_1>0$, $c_2<0$.

42. a) $f(3)=-1$. b) $f'(3)=\frac{1}{2}$. c) $f''(3)=-\frac{1}{6}$.

d) $\sum_{k=0}^{\infty}(-1)^{k+1}\frac{k!}{(2k)!}3^k(x-1)^k$ Substitute $3x$ into the series, then simplify.

43. $\frac{1}{12}$ Integrate term by term. The result is recognizable as the series for $\frac{x}{1-x}$.

44. $\cos(2\theta)=-\frac{1}{2}-\sqrt{3}\left(\theta-\frac{\pi}{3}\right)+\left(\theta-\frac{\pi}{3}\right)^2+\frac{2\sqrt{3}}{3}\left(\theta-\frac{\pi}{3}\right)^3+\cdots$

45. a) $\sin 1$ b) $\ln(1.5)$ c) Series diverges because $\frac{\pi}{e}>1$.

46. $-\frac{1}{11}$ Use the series for $\sin x$ to find the series for $\frac{\sin x}{x}$, then consider the term containing x^{11}.

47. a) $x\ln(1+2x)=2x^2-\frac{4x^3}{2}+\frac{8x^4}{3}-\frac{16x^5}{4}+\cdots=\sum_{n=0}^{\infty}\frac{(-1)^n 2^{n+1}x^{n+2}}{n+1}$

b) $e^{-x^2}=1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\cdots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{n!}$

48. $\frac{a}{(a+r)^2}=\frac{1}{a}\left(1+\frac{r}{a}\right)^{-2}=\frac{1}{a}\left(1-2\left(\frac{r}{a}\right)+3\left(\frac{r}{a}\right)^2-4\left(\frac{r}{a}\right)^3+\cdots\right)$

49. a) False b) False c) False

50. $f(x)=\int_0^x \tan^{-1}(t)dt=\int_0^x\left(t-\frac{t^3}{3}+\frac{t^5}{5}-\frac{t^7}{7}+\cdots\right)dt=\frac{x^2}{2}-\frac{x^4}{12}+\frac{x^6}{30}-\frac{x^8}{56}+\cdots$

The Taylor series for $\tan^{-1}(t)$ at $t=0$ would be given.

51. $Q=0$ stable, $Q=1$ unstable

53. a) *ii* b) *iii* c) *i* d) *iv*

54. $y(t)=e^{3/4}e^{-(1/4)t}$ or $y(t)=e$

55. a) $y(x)=\frac{1}{2}x\sqrt{4-x^2}+2\arcsin\left(\frac{x}{2}\right)-1-\frac{\sqrt{3}}{2}-\frac{\pi}{3}$

b) $x(\theta)=\frac{1}{2}\sin\theta\cos\theta+\frac{\theta}{2}+1-\frac{\pi}{2}$

c) $y(t)=-\frac{3}{2}e^{4t^2}+\frac{1}{2}$ Watch for the sign issues when you remove the absolute values.

56. a) $\frac{dQ}{dt}=-\alpha Q$ where $\alpha>0$ b) $Q(t)=Ae^{-\alpha t}$ c) $t=\frac{7\ln(90)}{\ln(9/5)}\approx 53.59$ hours

57. a) *ii* b) *iii* c) *i* d) *iv* e) *vi* f) *v*

58. a) $\frac{dL}{dt}=4-0.6L$, $L(t)=\frac{Ae^{-0.6t}+4}{0.6}$

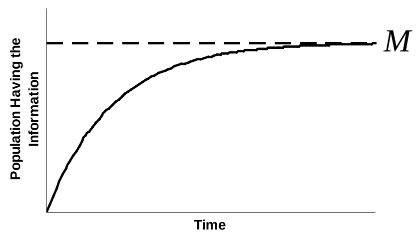
b) The stable equilibrium solution is $L=\frac{20}{3}$. If we start with 20/3 grams per square centimeter of leaves, we will always have that amount.

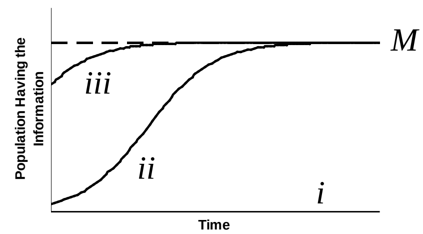
59. a) *i* b) *ii* c) *iii* d) *iv*

60. a) $r(t)=\frac{1}{15}t+60$ b) $\frac{dH}{dt}=k\left(H-\left(\frac{1}{15}t+60\right)\right)$ $H(0)=180$ where $k<0$

61. $\frac{dA}{dt}=0.06\sqrt{A}$, $A(t)=(0.03t+c)^2$

62. a) $\frac{dP}{dt}=k(M-P)$ where $k>0$ b) $\frac{dP}{dt}=kP(M-P)$ where $k>0$





63. a) $\int_0^{15} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ b) $\int_0^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

c) $\int_0^{10} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ d) $\int_3^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

64. $500\cdot 45+\int_0^{45} 3(45-x)dx=25{,}537.5$ foot-pounds One possibility.

65. $\int_0^{10} 9.8\left(\frac{8}{0.5\cdot 24\cdot 10}\right)(10-h)\left(\frac{12}{5}h\right)dh$ Joules One possibility.

66. $\int_0^{40} 62.4(40-h)200dh$ pounds One possibility.