MATH 129
FINAL EXAM REVIEW PACKET ANSWERS
(Fall 2013)

1. $\int_0^3 \left(10\pi\sin\left(\frac{\pi}{3}t\right)+30\right)dt = 150$ people

2. $\int_1^2 f(5-2x)dx = \frac{7}{2}$ Let $u = 5-2x$ and change the endpoints.

3. a) $\int \frac{t}{\sqrt{t+1}}dt = \frac{2}{3}(t+1)^{3/2} - 2(t+1)^{1/2} + c$ By the method of substitution with $u = t+1$.
You can also use integration by parts with $u = t$ and $v' = (t+1)^{-1/2}$. The result is equivalent, just written in a different form. $\int \frac{t}{\sqrt{t+1}}dt = 2t(t+1)^{1/2} - \frac{4}{3}(t+1)^{3/2} + c$

b) $\int \left(\frac{1}{z^2}+A\right)^2 dz = -\frac{1}{3z^3} - \frac{2A}{z} + A^2 z + c$ Distribute.

c) $\int 3^x e^x dx = \frac{1}{\ln(3e)}(3e)^x + c$ Rewrite $3^x e^x = (3e)^x$.

4. a) $\int \frac{\ln\left(z^2+1\right)}{z^2}dz = -\frac{1}{z}\ln\left(z^2+1\right) + 2\arctan z + c$ Let $u = \ln(z^2+1)$ and $v' = \frac{1}{z^2}$.

b) $\int x\arcsin(x^2)dx = \frac{1}{2}x^2\arcsin(x^2) + \frac{1}{2}\sqrt{1-x^4} + c$ First make a substitution with $w = x^2$.
Then let $u = \arcsin(w)$ and $v' = 1$.

c) $\int_0^1 x \cdot g''(x)dx = 3$ Let $u = x$ and $v' = g''$ for the first integration by parts.

5. a) $\int \cos^2(3\theta+2)d\theta = \frac{1}{6}\cos(3\theta+2)\sin(3\theta+2) + \frac{1}{6}(3\theta+2) + c$ Let $u = 3\theta+2$ before using table formula # 18. If you use another approach, your answer will look different.

b) $\int \frac{2}{4t^2-9}dt = \frac{1}{6}\left(\ln|2t-3| - \ln|2t+3|\right) + c$ Let $u = 2t$ and factor the denominator before using table formula # 26. If you use another approach, your answer will look different.

c) $\int \frac{dy}{\sqrt{y^2+8y+15}} = \ln\left|(y+4)+\sqrt{y^2+8y+15}\right|+c$ Complete the square before using table formula # 29.

d) $\int \frac{\sin(4\alpha)}{\cos^2(4\alpha)-\cos(4\alpha)}\,d\alpha = \frac{1}{4}\left(\ln\left|\cos(4\alpha)\right|-\ln\left|\cos(4\alpha)-1\right|\right)+c$ Let $u=\cos(4\alpha)$ and factor the denominator before using table formula # 26.

6. $\int \frac{3y^3+5y-1}{y^3+y}dy = 3y-\ln|y|+\frac{1}{2}\ln\left|y^2+1\right|+2\arctan(y)+c$ First do long division, then use partial fractions $\frac{A}{y}+\frac{By+C}{y^2+1}$.

7. $\int \frac{dx}{(5-x^2)^{3/2}} = \frac{1}{5}\frac{x}{\sqrt{5-x^2}}+c$ Let $x=\sqrt{5}\sin(\theta)$.

8. $v=k(R^2-r^2)$. $\frac{1}{R-0}\int_0^R k(R^2-r^2)dr = \frac{2}{3}kR^2$

9. $E(X)=\int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} xf(x)dx+\int_0^{\infty} xf(x)dx = 0+\int_0^{\infty} x\frac{1}{7}e^{-x/7}dx = 7$
Use integration by parts or the table of integrals. Remember to use proper notation.

10. $\int_0^1 e^{-t^2}dt \approx \frac{1}{2}e^{-1/16}+\frac{1}{2}e^{-9/16} \approx 0.75459794$

11. a), d), and e)

12. Trap, Right, Mid, and Left

13. a) The integral converges. $\int_0^{\infty}\frac{1}{x^2+4}dx = \frac{\pi}{4}$ Use table formula # 24.

b) The integral converges. $\int_1^{\infty}\frac{1}{2^x}dx = \frac{1}{2\ln 2}$.

c) The integral diverges. $\int_0^1 \frac{e^x}{\left(e^x-1\right)^2}dx = \infty$ Let $u=e^x-1$.

d) The integral converges. $\int_{\pi/6}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} dx = 2\sqrt{\frac{\sqrt{3}}{2}}$ Let $u = \cos x$

e) The integral diverges. $\lim_{x\to\infty} \frac{x^5}{e^{-x}+x} = \infty$. In order for the improper integral to converge, the integrand must approach 0.

14. $\int_m^{\infty} e^{-\left(\frac{x-m}{s}\right)^2} dx = \frac{s\sqrt{\pi}}{2}$ Let $u = \frac{x-m}{s}$ and change the endpoints.

15. a) The integral converges. Rewrite as $\int_0^{\infty} a \cdot f(x)dx = a\int_0^{\infty} f(x)dx$.

b) The integral converges. Let $u = ax$.

c) The integral diverges. Rewrite as $\int_0^{\infty} (a + f(x))dx = \int_0^{\infty} adx + \int_0^{\infty} f(x)dx$.

d) The integral converges. Let $u = a + x$.

16. a) The integral converges. $\int_2^{\infty} \frac{d\theta}{\sqrt{\theta^3+2}} < \sqrt{2}$ Use the comparison $\frac{1}{\sqrt{\theta^3+2}} < \frac{1}{\theta^{3/2}}$.

b) The integral converges. $\int_1^{\infty} \frac{1+\sin^2 x}{(x+3)^3} dx < \frac{1}{16}$ Use the comparison $\frac{1+\sin^2 x}{(x+3)^3} < \frac{2}{(x+3)^3}$.

c) The integral diverges. $\int_1^{\infty} \frac{(1+\sin^2 x)x^2}{x^3+3} dx$ Use the comparison $\frac{(1+\sin^2 x)x^2}{x^3+3} > \frac{x^2}{x^3+3}$.

17. b) and c)

18. $\int_0^6 16\sqrt{36-(6-x)^2}\,dx$ Use Pythagorean Theorem to find the width of the slice.

19. a) $\int_0^1 3\sqrt{x}dx + \int_1^2 (6-3x)dx$ b) $\int_0^3 \left(\frac{6-y}{3} - \frac{y^2}{9}\right) dy$

20. a) $75\pi - \int_0^3 \pi(5e^{-x})^2 dx = \frac{(125+25e^{-6})}{2}\pi$ b) $\int_0^3 \pi\left(5-5e^{-x}\right)^2 dx = \frac{(75+100e^{-3}-25e^{-6})\pi}{2}$

21. a) $\int_0^8 \pi\left(y^{1/3}\right)^2 dy = \frac{96}{5}\pi$ b) $\int_0^8 \pi\left(y^{1/3}+2\right)^2 dy - 32\pi = \frac{336}{5}\pi$

22. $\int_0^2 \left(\pi\left(f(x)\right)^2 - \pi\left(g(x)\right)^2\right)dx + \int_2^7 \left(\pi\left(g(x)\right)^2 - \pi\left(f(x)\right)^2\right)dx$

23. a) $\int_0^\pi (\sin x)^2 dx = \frac{\pi}{2}$ b) $\int_0^\pi \frac{1}{2}\pi\left(\frac{\sin x}{2}\right)^2 dx = \frac{\pi^2}{16}$

24. Left hand rule:

$$10\cdot\pi\left(\frac{26}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{22}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{18}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{12}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{6}{2\pi}\right)^2 = \frac{4160}{\pi} \text{ cubic inches}$$

Right hand rule:

$$10\cdot\pi\left(\frac{22}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{18}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{12}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{6}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{2}{2\pi}\right)^2 = \frac{2480}{\pi} \text{ cubic inches}$$

Trapezoid rule: $\frac{3320}{\pi}$ cubic inches Use the average of the left and right hand rules.

25. $10^3\int_0^5 2\pi r(0.115)e^{-2r}dr = 10^3 \cdot 0.0575\pi\left(1-11e^{-10}\right)$
Approximately 180.55 cubic meters of soot. (Note the change in units throughout).

26. $\int_0^5 \left(2+0.5\cosh x\right)dx = 10+0.5\sinh 5$ grams

27. a) $\int_0^8 \delta(x)\cdot 2\pi x dx$ b) $\int_{-8}^8 \delta(x)\cdot 2\sqrt{64-x^2}\,dx$

28. $\int_0^{25}\left(-\frac{8}{5}h+90\right)\pi(10)^2 dh = 175{,}000\pi$ pounds

29. a) $a_n = \frac{(-1)^n 2n}{(n+2)^2}$ b) $\lim_{n\to\infty} a_n = -\frac{3}{5}$ $\lim_{n\to\infty} b_n = 5$

30. a) $\dfrac{(3/64)\left(1-(1/4)^8\right)}{1-1/4}=\dfrac{65535}{1048576}$ b) $\dfrac{3/4}{1-1/4}=1$ c) $2(80)-5(43)=-55$

31. $P_n=\dfrac{(0.05)(200)\left(1-0.05^{n-1}\right)}{1-0.05}$ $Q_n=\dfrac{(200)\left(1-0.05^{n}\right)}{1-0.05}$ $n=1,2,3,\cdots$

32. a) The series converges. $\int_2^\infty \dfrac{1}{x(\ln x)^2}dx=\dfrac{1}{\ln 2}$ Use the method of section 7.7 to evaluate the improper integral.

b) The series diverges. $\int_1^\infty \dfrac{3x^2+2x}{\sqrt{x^3+x^2+1}}dx=\infty$ Use the method of section 7.7 to evaluate the improper integral.

33. a) The series diverges. $\lim_{n\to\infty}\dfrac{e(n)^2}{2(n+1)^2}=\dfrac{e}{2}>1$

b) The series converges. $\lim_{n\to\infty}\dfrac{(n+1)^2}{(2n+2)(2n+1)}=\dfrac{1}{4}<1$

34. The Ratio Test gives us 1 and does not tell us anything about the convergence or divergence of the series.

35. The given series is conditionally convergent. $\sum_{k=5}^{\infty}\dfrac{(-1)^{k-1}}{k(\ln k)}$ converges by the Alternating Series Test, while $\sum_{k=5}^{\infty}\left|\dfrac{(-1)^{k-1}}{k(\ln k)}\right|$ diverges by the Integral Test.

36. a) and b)

37. a) False b) True c) False d) True e) False

38. a) The radius of convergence is $R=3$. The interval of convergence is $(-7,-1)$.
b) The radius of convergence is $R=\infty$. The interval of convergence is $(-\infty,\infty)$.
c) The radius of convergence is $R=0$. The series only converges for $x=1$.

39. a) True b) True c) Impossible to determine.

40. $P_2(x) = 4 + \frac{1}{3}(x-1) - \frac{1}{144}(x-1)^2$ $f(2) \approx P_2(2) = \frac{623}{144} \approx 4.3264$

41. The sign of c_0 cannot be determined, $c_1 > 0$, $c_2 < 0$.

42. a) $f(3) = -1$. b) $f'(3) = \frac{1}{2}$. c) $f''(3) = -\frac{1}{6}$.

d) $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{k!}{(2k)!} 3^k (x-1)^k$ Substitute $3x$ into the series, then simplify.

43. $\frac{1}{12}$ Integrate term by term. The result is recognizable as the series for $\frac{x}{1-x}$.

44. a) *ii* b) *iv* c) *i* d) *iii*

45. a) $\sin 1$ b) $\ln(1.5)$ c) Series diverges because $\frac{\pi}{e} > 1$.

46. $-\frac{1}{11}$ Use the series for $\sin x$ to find the series for $\frac{\sin x}{x}$, then consider the term containing x^{11}.

47. a) $x\ln(1+2x) = 2x^2 - \frac{4x^3}{2} + \frac{8x^4}{3} - \frac{16x^5}{4} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{n+2}}{n+1}$

b) $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

48. $\frac{a}{(a+r)^2} = \frac{1}{a}\left(1+\frac{r}{a}\right)^{-2} = \frac{1}{a}\left(1 - 2\left(\frac{r}{a}\right) + 3\left(\frac{r}{a}\right)^2 - 4\left(\frac{r}{a}\right)^3 + \cdots\right)$

49. a) False b) False c) False

50. a) $\frac{\sqrt{3}}{2} - \frac{1}{2}i = e^{-(\pi/6)i}$ b) $2e^{\frac{-\pi}{4}i} = \sqrt{2} - i\sqrt{2}$ c) $e^{(3+4i)t} = e^{3t}\cos(4t) + i \cdot e^{3t}\sin(4t)$

d) $2^9 i = 512i$ Write $1+i$ in the form $Re^{i\theta}$, then use rules for exponents.

51. a) $7+24i$ b) $\frac{3}{29}-\frac{22}{29}i$ c) $\frac{\frac{1}{2}+\frac{\sqrt{3}}{2}i}{2-i}=\frac{2-\sqrt{3}}{10}+\frac{1+2\sqrt{3}}{10}i$

52. a) $\left(e^{i\theta}\right)^3=(\cos\theta+i\sin\theta)^3=\cos^3\theta+3i\cos^2\theta\sin\theta-3\cos\theta\sin^2\theta-i\sin^3\theta$

b) $\left(e^{i\theta}\right)^3=e^{i3\theta}=\cos(3\theta)+i\sin(3\theta)$

c) $\cos(3\theta)=\cos^3\theta-3\cos\theta\sin^2\theta$ and $\sin(3\theta)=-\sin^3\theta+3\cos^2\theta\sin\theta$

53. a) *ii* b) *iii* c) *i* d) *iv*

54. $y(t)=e^{3/4}e^{-(1/4)t}$ or $y(t)=e$

55. a) $y(x)=\frac{1}{2}x\sqrt{4-x^2}+2\arcsin\left(\frac{x}{2}\right)-1-\frac{\sqrt{3}}{2}-\frac{\pi}{3}$

b) $x(\theta)=\frac{1}{2}\sin\theta\cos\theta+\frac{\theta}{2}+1-\frac{\pi}{2}$

c) $y(t)=-\frac{3}{2}e^{4t^2}+\frac{1}{2}$ Watch for the sign issues when you remove the absolute values.

56. a) $\frac{dQ}{dt}=-\alpha Q$ where $\alpha>0$ b) $Q(t)=Ae^{-\alpha t}$ c) $t=\frac{7\ln(90)}{\ln(9/5)}\approx 53.59$ hours

57. a) *ii* b) *iii* c) *i* d) *iv* e) *vi* f) *v*

58. a) $\frac{dL}{dt}=4-0.6L$, $L(t)=\frac{Ae^{-0.6t}+4}{0.6}$

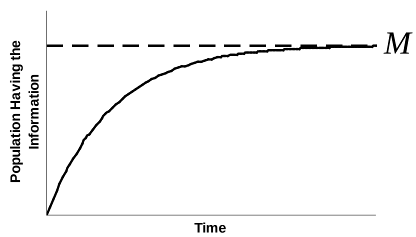
b) The stable equilibrium solution is $L=\frac{20}{3}$. If we start with 20/3 grams per square centimeter of leaves, we will always have that amount.

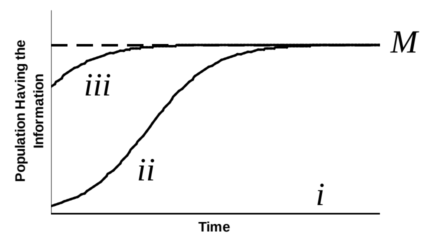
59. a) *i* b) *ii* c) *iii* d) *iv*

60. a) $r(t)=\frac{1}{15}t+60$ b) $\frac{dH}{dt}=k\left(H-\left(\frac{1}{15}t+60\right)\right)$ $H(0)=180$ where $k<0$

61. $\frac{dA}{dt}=0.06\sqrt{A}$, $A(t)=(0.03t+c)^2$

62. a) $\frac{dP}{dt}=k(M-P)$ where $k>0$ b) $\frac{dP}{dt}=kP(M-P)$ where $k>0$





63. a) $\int_0^{15} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ b) $\int_0^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

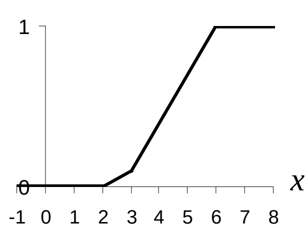
c) $\int_0^{10} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ d) $\int_3^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

64. $500\cdot 45+\int_0^{45} 3(45-x)dx=25{,}537.5$ foot-pounds

65. $\int_0^{10}\left(\frac{240}{0.5\cdot 24\cdot 10}\right)(10-h)\left(\frac{12}{5}h\right)dh=800$ foot-pounds

66. a) Probability density function. $c=\frac{1}{10}$.

b) The cumulative distribution function.



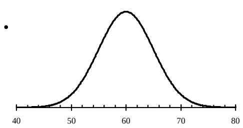
67. a) $\int_2^3 \frac{4}{81} t^3 dt = \frac{65}{81}$ or approximately 80.2%

b) $P(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{81} t^4 & 0 \le t \le 3 \\ 1 & t > 3 \end{cases}$

68. The probability that the time between successive calls is between 5 and 10 minutes:
$F(10) - F(5) = e^{-1} - e^{-2} \approx 0.23$

69. a) Both have units of mph.

b) The maximum occurs at $x = 60$. Inflection points are at $x = 55$ and $x = 65$.



c) 0.5

d) 0.16   Use symmetry of the graph.