Homework 7 Solutions

42.

$$\begin{aligned} \log x + \log(x+3) &= 1 \\ \log x(x+3) &= 1 \\ &\Updownarrow \\ 10^1 &= x(x+3) \\ 10 &= x^2 + 3x \\ 0 &= x^2 + 3x - 10 \\ 0 &= (x+5)(x-2). \end{aligned}$$

Thus, we have two possibilities, $x = -5$ or $x = 2$, however, the domain of the logarithmic function is the set of positive real numbers, so $\log(-5)$ does not exist! Therefore the only solution is $x = 2$.

54. We want to find the inverse function $f^{-1}(x)$, so we have

$$\begin{aligned} f(x) &= 5^{x+3} \\ y &= 5^{x+3} \quad \text{now swap the } x\text{'s and } y\text{'s then solve for } y \\ x &= 5^{y+3} \\ \log x &= \log 5^{y+3} \\ \log x &= (y+3)\log 5 \\ \frac{\log x}{\log 5} &= y+3 \\ \frac{\log x}{\log 5} - 3 &= y \\ \frac{\log x}{\log 5} - 3 &= f^{-1}(x). \end{aligned}$$

Note, the above could also be written (using the change of base formula) $f^{-1}(x) = -3 + \log_5 x$. Now, you should also check to be sure that $f^{-1}(x)$ actually is the inverse of $f(x)$ by showing $f^{-1} \circ f(x) = x$ and $f \circ f^{-1}(x) = x$.