Homework 6 Solutions

4. To determine the value of each logarithm exactly, we want to solve each of the below equations for x:

   (a) $\log_2 8\sqrt{2}$

$$\begin{aligned} \log_2 8\sqrt{2} &= x \\ 2^x &= 8\sqrt{2} \\ 2^x &= 2^3 \cdot 2^{1/2} \\ 2^x &= 2^{3+1/2} \\ 2^x &= 2^{7/2}. \end{aligned}$$

   So we see $x = \frac{7}{2}$, and therefore $\log_2 8\sqrt{2} = \frac{7}{2}$.

   (b) $\log_{(2/5)} \left(\frac{4}{25}\right)$

$$\begin{aligned} \log_{(2/5)} \left(\frac{4}{25}\right) &= x \\ \left(\frac{2}{5}\right)^x &= \frac{4}{25} \\ \left(\frac{2}{5}\right)^x &= \left(\frac{2}{5}\right)^2. \end{aligned}$$

   So we see $x = 2$, and therefore $\log_{(2/5)} \left(\frac{4}{25}\right) = 2$.

   (c) By the properties of logarithms we know $\ln e = 1$.

   (d) We know the domain of the logarithmic functions is the set of positive real numbers, hence $\log_4(-2)$ does not exists since -2 is not a positive real number!

20. (a)

$$\begin{aligned} \log_2(k-3) - 3\log_2(k+5) &= \log_2(k-3) - \log_2(k+5)^3 \\ &= \log_2 \left(\frac{k-3}{(k+5)^3}\right). \end{aligned}$$

   (b)

$$\begin{aligned} \log_5(z-3) - \log_5 x + \log_5(2y) &= \log_5 \left(\frac{z-3}{x}\right) + \log_5(2y) \\ &= \log_5 \left(\frac{2y(z-3)}{x}\right) \end{aligned}$$