Homework 5 Solutions

20. We want to determine $P^{-1}(x)$, so notice

$$\begin{aligned} P(x) &= (x-6)^3 \\ y &= (x-6)^3 \quad \text{(now switch the positions of } x \text{ and } y \text{ and solve for } y\text{)} \\ x &= (y-6)^3 \\ \sqrt[3]{x} &= y-6 \\ \sqrt[3]{x}+6 &= y \\ \sqrt[3]{x}+6 &= P^{-1}(x). \end{aligned}$$

You should check to be sure that $P^{-1}(x)$ actually is the inverse of $P(x)$ by showing $P^{-1} \circ P(x) = x$ and $P \circ P^{-1}(x) = x$.

32. We want to determine $f^{-1}(x)$, so notice

$$\begin{aligned} f(x) &= \frac{7-3x}{5-4x} \\ y &= \frac{7-3x}{5-4x} \quad \text{(now switch the positions of } x \text{ and } y \text{ and solve for } y\text{)} \\ x &= \frac{7-3y}{5-4y} \\ x(5-4y) &= 7-3y \\ 5x-4yx &= 7-3y \\ 5x-7 &= 4yx-3y \\ 5x-7 &= y(4x-3) \\ \frac{5x-7}{4x-3} &= y \\ \frac{5x-7}{4x-3} &= f^{-1}(x). \end{aligned}$$

You should check to be sure that $f^{-1}(x)$ actually is the inverse of $f(x)$ by showing $f^{-1} \circ f(x) = x$ and $f \circ f^{-1}(x) = x$.