Homework 2 Solutions

Section 1.4:

15. (a) First, we know the perimeter of the fence is to be 280 yards and we do not need fence along one of our lengths, ℓ, so it follows

$$\text{Perimeter} = 280 = \ell + 2w.$$

Using this, we can see that $\ell = 280 - 2w$ (by solving the above equation for ℓ). Substituting this into the equation for area, we see

$$\begin{aligned} \text{Area} &= \ell \cdot w \\ &= (280 - 2w) \cdot w \\ &= 280w - w^2. \end{aligned}$$

Since we know $\ell = 280 - 2w$ it follows that w must be less than 140 yards, or else ℓ will be negative (or 0). Hence, the domain of our function is $(0, 140)$.

(b) The x-intercepts are 0 and 140, and the y-intercept is 0. The function is increasing on $(0, 70)$ and decreasing on $(70, 140)$.

(c) Dimensions giving maximum area: 70×140. So maximum area is 9800 square yards.

18. (a) First, since the base of the box is a square, we have that the length of the base equals the width of the base – i.e. $\ell = w$. Now, we know the area is 6 cubic feet, so

$$\begin{aligned} \text{Volume} = 6 &= \ell \cdot w \cdot h \\ &= w \cdot w \cdot h \\ &= w^2 h. \end{aligned}$$

We can solve this equation for w to find that

$$w^2 = \frac{6}{h}$$

and

$$w = \frac{\sqrt{6}}{\sqrt{h}}.$$

Next, we must find a formula for the total cost of constructing the box. So, we know

$$\begin{aligned} (\text{Total Cost}) &= (\text{Cost of Base}) + (\text{Cost of 4 Sides}) \\ &= (\text{Cost of Base}) + 4(\text{Cost of 1 Side}) \\ &= (3w^2) + 4(2hw) \\ &= 3\left(\frac{6}{h}\right) + 8h\left(\frac{\sqrt{6}}{\sqrt{h}}\right) \\ &= \frac{18}{h} + 8\sqrt{6h}. \end{aligned}$$

Hence, our total cost function is $TC(h) = \frac{18}{h} + 8\sqrt{6h}$.

(b) The domain of our cost function is $h > 0$. And we see

$$\begin{aligned} TC(1) &= \frac{18}{1} + 8\sqrt{6 \cdot 1} \approx 37.6 \\ TC(2) &= \frac{18}{2} + 8\sqrt{6 \cdot 2} \approx 36.7, \end{aligned}$$

so it is more expensive to construct a box of height 1.

(c) The dimensions giving the minimum cost will be $2 \times 2 \times 1.5$, with a minimum cost of \$36.