Homework 6 Solutions

Section 4.4:

4. To determine the value of each logarithm exactly, we want to solve each of the below equations for x:

   (a) $\log_2 8\sqrt{2}$

$$\begin{aligned} \log_2 8\sqrt{2} &= x \\ 2^x &= 8\sqrt{2} \\ 2^x &= 2^3 \cdot 2^{1/2} \\ 2^x &= 2^{3+1/2} \\ 2^x &= 2^{7/2}. \end{aligned}$$

   So we see $x = \frac{7}{2}$, and therefore $\log_2 8\sqrt{2} = \frac{7}{2}$.

   (b) $\log_{(2/5)} \left(\frac{4}{25}\right)$

$$\begin{aligned} \log_{(2/5)} \left(\frac{4}{25}\right) &= x \\ \left(\frac{2}{5}\right)^x &= \frac{4}{25} \\ \left(\frac{2}{5}\right)^x &= \left(\frac{2}{5}\right)^2. \end{aligned}$$

   So we see $x = 2$, and therefore $\log_{(2/5)} \left(\frac{4}{25}\right) = 2$.

   (c) By the properties of logarithms we know $\ln e = 1$.

   (d) We know the domain of the logarithmic functions is the set of positive real numbers, hence $\log_4(-2)$ does not exists since -2 is not a positive real number!

20. (a)

$$\begin{aligned} \log_2(k-3) - 3\log_2(k+5) &= \log_2(k-3) - \log_2(k+5)^3 \\ &= \log_2\left(\frac{k-3}{(k+5)^3}\right). \end{aligned}$$

   (b)

$$\begin{aligned} \log_5(z-3) - \log_5 x + \log_5(2y) &= \log_5\left(\frac{z-3}{x}\right) + \log_5(2y) \\ &= \log_5\left(\frac{2y(z-3)}{x}\right) \end{aligned}$$

Section 4.5

42.

$$\begin{aligned} \log x + \log(x+3) &= 1 \\ \log x(x+3) &= 1 \\ &\Updownarrow \\ 10^1 &= x(x+3) \\ 10 &= x^2+3x \\ 0 &= x^2+3x-10 \\ 0 &= (x+5)(x-2). \end{aligned}$$

Thus, we have two possibilities, $x=-5$ or $x=2$, however, the domain of the logarithmic function is the set of positive real numbers, so $\log(-5)$ does not exist! Therefore the only solution is $x=2$.

54. We want to find the inverse function $f^{-1}(x)$, so we have

$$\begin{aligned} f(x) &= 5^{x+3} \\ y &= 5^{x+3} \quad \text{now swap the } x\text{'s and } y\text{'s then solve for } y \\ x &= 5^{y+3} \\ \log x &= \log 5^{y+3} \\ \log x &= (y+3)\log 5 \\ \frac{\log x}{\log 5} &= y+3 \\ \frac{\log x}{\log 5} - 3 &= y \\ \frac{\log x}{\log 5} - 3 &= f^{-1}(x). \end{aligned}$$

Note, the above could also be written (using the change of base formula) $f^{-1}(x) = -3+\log_5 x$. Now, you should also check to be sure that $f^{-1}(x)$ actually is the inverse of $f(x)$ by showing $f^{-1}\circ f(x) = x$ and $f \circ f^{-1}(x) = x$.