Homework 1 Solutions

Section 1.1:

12. $C(x) = \dfrac{\sqrt{2x^2+7}}{x^2+3}$

(a) Check when the denominator is zero:

$$\begin{aligned} x^2+3 &= 0 \\ x^2 &= -3 \\ x &= \pm\sqrt{-3}. \end{aligned}$$

This does not exist, so the denominator is never zero!
Check for taking square roots of negative numbers:

$$\begin{aligned} \sqrt{2x^2+7} &< 0 \\ (\sqrt{2x^2+7})^2 &< 0^2 \\ 2x^2+7 &< 0 \\ 2x^2 &< -7 \\ x^2 &< -\frac{7}{2} \\ x &< \pm\sqrt{-\frac{7}{2}}. \end{aligned}$$

This does not exist, so there are never square roots of negative numbers!
Since the denominator is never zero, and there are no square roots of negative numbers, the domain of C is all real numbers (or $(-\infty, \infty), \mathbb{R}$).

(b) To find the zeros of C, we solve the equation $C(x) = 0$:

$$\begin{aligned} \frac{\sqrt{2x^2+7}}{x^2+3} &= 0 \\ \sqrt{2x^2+7} &= 0(x^2+3) \\ \sqrt{2x^2+7} &= 0 \\ (\sqrt{2x^2+7})^2 &= 0^2 \\ 2x^2+7 &= 0 \\ 2x^2 &= -7 \\ x^2 &= \frac{-7}{2} \\ x &= \pm\sqrt{\frac{-7}{2}}. \end{aligned}$$

Since $\frac{-7}{2} < 0$, we cannot take its square root. Hence C has no zeros!

16. $T(x) = \dfrac{x+1}{2x}$

(a) $T(y-2) = \dfrac{(y-2)+1}{2(y-2)} = \dfrac{y-1}{2y-4}$

(b) $T(2-y) = \dfrac{(2-y)+1}{2(2-y)} = \dfrac{3-y}{4-2y}$

(c) $y - T(2) = y - \dfrac{(2)+1}{2(2)} = y - \dfrac{3}{4}$

(d) $T(y) - T(2) = \dfrac{(y)+1}{2(y)} - \dfrac{(2)+1}{2(2)} = \dfrac{y+1}{2y} - \dfrac{3}{4} = \dfrac{2(y+1)}{2(2y)} - \dfrac{3(y)}{4(y)} = \dfrac{2y+2-3y}{4y} = \dfrac{2-y}{4y}$

32. Recall,

$$n(x) = \begin{cases} -\sqrt{2} & x < -3 \\ x+5 & -3 \le x \le 3 \\ x^2 - 4 & x > 3 \end{cases}$$

(a) $n(-4) = -\sqrt{2}$

(b) $n(-3) = (-3) + 5 = 2$

(c) $n(3) = (3) + 5 = 8$

(d) $n(6) = (6)^2 - 4 = 36 - 4 = 32$

Section 1.2

2.
- $g(-5) = -2$
- $g(-1) = 2$
- $g(2) = -1$
- $g(5) = 2$
- $g(7) = 6$

8. (a) Turning points of g: $(-1, 2)$ and $(3, -2)$.

(b) Increasing on: $(-5, -1) \cup (3, 7)$
Decreasing on: $(-1, 3)$

36. Let $L(x) = (4 - x^2)(2 + x^2)$

(a) Determine the x-intercepts:

$$L(x) = (4 - x^2)(2 + x^2) = 0$$

Setting each term equal to zero, we have $(4 - x^2) = 0$ implying $x = \pm 2$, and $(2 + x^2) = 0$ never happens since 2 and x^2 are always bigger than zero. hence our only x-intercepts are $(-2, 0)$ and $(2, 0)$.

(b) Determine the intervals over which the function is positive and the intervals over which it is negative.

- Positive on: $(-2, 2)$
- Negative on: $(-\infty, -2) \cup (2, \infty)$

As seen from the graph and using the x-intercepts from part (a).

(c) Determine the open intervals over which the function is increasing and the open intervals over which it is decreasing.

- Increasing on: $(-\infty, -1) \cup (0, 1)$
- Decreasing on: $(-1, 0) \cup (1, \infty)$

As seen from the graph.