Final Exam Study Aid

Note: This study aid is intended to help you review for the final exam. It covers the primary concepts in the course, with a large emphasis on the second half (post midterm) of the material. You should check the Midterm Study Aid for more review on the earlier material. Although the final exam will be similar to the study aid, it will not be identical to it. You should also review tests, notes, study aids and homework given during the semester. The formulas given below will be identical to the formulas given on the final exam.

FORMULAS

$$A = P\left(1 + \frac{r}{n}\right)^{n\,t} \qquad A = Pe^{r\,t}$$

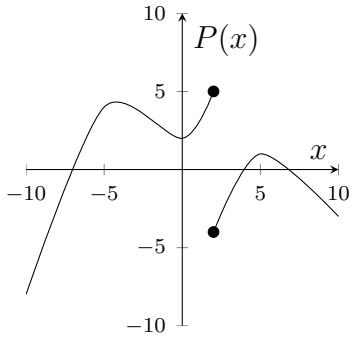
1. Which of the following represent y as a function of x?

(1)

x	y
-2	8
0	-1
1	4
3	2
5	-1
1	4

(2) $x^2 + y^2 = 5$

(3)



(A) 1 and 3 only (B) 1 only (C) 3 only

(D) 2 only (E) None of them

2. Identify the equations that determine y as a function of x:

(1) $x^2 + y^2 - 4 = 0$ (2) $3y^3 + 2x = 7$ (3) $x^2y + 5y = -2$

(A) 1 and 3 only (B) 2 only (C) 2 and 3 only

(D) All of them (E) None of them

Use the function $h(x) = \begin{cases} 2x-5 & \text{for } x \leq -1 \\ 5 & \text{for } -1 < x \leq 4 \\ -x^2 & \text{for } x > 4 \end{cases}$ to answer the next three questions.

3. Evaluate $h(-1)$

   (A) -7 (B) 5 (C) -3 (D) -1 (E) None of these

4. Evaluate $h(3) + h(6)$

   (A) 41 (B) -81 (C) -21 (D) -31 (E) 51

5. Evaluate $h(h(2))$

   (A) 25 (B) 5 (C) -25 (D) 100 (E) -100

6. A couple invests \$3500 to build a rose garden. On the average, it costs them \$0.35 to grow each rose. If each rose can be sold for \$1.75, how many roses must they sell to break even?

   (A) 2,000 (B) 2,500 (C) 10,000 (D) 8,250 (E) None of these

7. A certain class has 9 quizzes throughout the semester. A student has an average of A on the first six quizzes. If the student then has an average of B on the last three quizzes, determine the students average on all 9 quizzes.

   (A) $\frac{A+B}{2}$ (B) $\frac{6A+3B}{2}$ (C) $\frac{A+B}{9}$ (D) $\frac{6A+3B}{9}$ (E) None of these

8. The conversion rate between kilograms and pounds is about 2.2 pounds per kilogram. That is, if a person weighs 100 kilograms, they would weigh about 220 pounds. Let k represent the weight of an object in kilograms, and let p represent the weight in pounds. Express k as a function of p.

(A) $k(p) = 2.2 + p$ (B) $p(k) = \dfrac{k}{2.2}$ (C) $k(p) = 2.2p$

(D) $p(k) = 2.2k$ (E) $k(p) = \dfrac{p}{2.2}$

9. An above ground pool is in the shape of a circular cylinder with a radius of 12 feet. A hose begins filling the pool at a rate of 2 cubic feet per minute. Express the height of the water in the pool as a function of time t, in minutes.

(A) $h(t) = \dfrac{12t}{\pi}$ (B) $h(t) = \dfrac{t}{72\pi}$ (C) $h(t) = \dfrac{t}{144\pi}$ (D) $h(t) = \dfrac{2t}{\pi}$ (E) $h(t) = 144\pi t$

Consider the following problem: In 1990 Joe bought a BMW for \$40,000; it depreciates \$4,000 each year. Tom bought a classic '57 Chevy for \$10,000 and it appreciates \$1,000 each year. Use this information to answer the next TWO questions.

10. Write a system of equations for the values (V) of these cars in terms of the number of years (t). [$t = 0$ is 1990]

(A) BMW : $V = 40,000 - 4000t$
Chevy: $V = 10,000 + 1000t$

(B) BMW: $V = 40,000 + 4000t$
Chevy: $V = 10,000 - 1000t$

(C) BMW : $V = 40,000 - t$
Chevy: $V = 10,000 + t$

(D) BMW: $V = t + 40,000$
Chevy: $V = t + 10,000$

(E) None of these

11. How many years must pass before the values of both cars are equal? The answer is a number:

(A) between 1 and 5 (B) between 5 and 10 (C) more than 10

(D) never equal (E) None of these

12. The graph of $y = -g(x+5)$ can be produced by transforming the graph of $y = g(x)$ in which of the following ways?

(A) Horizontally sifting to the right 5 units and reflecting across the x-axis.

(B) Horizontally sifting to the left 5 units and reflecting across the y-axis.

(C) Horizontally sifting to the right 5 units and reflecting across the y-axis.

(D) Horizontally sifting to the left 5 units and reflecting across the x-axis.

(E) None of these

13. Determine functions $f(x)$ and $g(x)$ so that $(f \circ g)(x) = \dfrac{1}{(x-2)^2}$. Which of the following could be choices for $f(x)$ and $g(x)$.

(1) $f(x) = \dfrac{1}{x}$ and $g(x) = (x-2)^2$

(2) $f(x) = \dfrac{1}{x^2}$ and $g(x) = (x-2)$

(3) $f(x) = \dfrac{1}{x-2}$ and $g(x) = x^2$

(4) $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{1}{x-2}$

(A) 2 and 4 only (B) 1, 3, and 4 only (C) 1 and 2 only

(D) 1, 2, and 3 only (E) 1, 2, and 4 only

14. The domain of $f(x)$ is $[-5, 4]$ and the domain of $g(x)$ is $[-2, 6]$. The function $g(x)$ has zeroes at $x = 1$ and $x = 5$. What is the domain of $\left(\dfrac{f}{g}\right)(x)$?

(A) $\left[-\dfrac{5}{2}, \dfrac{2}{3}\right]$ (B) $[-5, 6]$ (C) $[-2, 4]$

(D) $[-2, 1) \cup (1, 4]$ (E) $[-5, 1) \cup (1, 5) \cup (5, 6]$

15. If $(2, -5)$ is a point on the graph of $r(x)$, which of the following points MUST be on the graph of $y = \frac{1}{3}r(x+1)$?

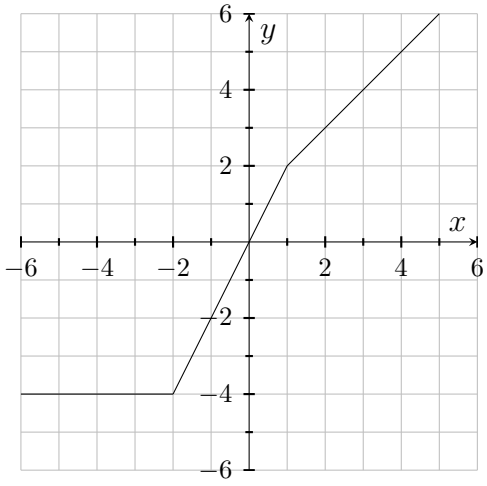
(A) $\left(\frac{2}{3}, -4\right)$ (B) $(1, -5)$ (C) $\left(3, -\frac{5}{3}\right)$ (D) $\left(\frac{1}{3}, -5\right)$ (E) $\left(1, -\frac{5}{3}\right)$

16. Given $f(x) = \log_4(-8x)$ and $h(x) = x - 3$, find $(h \circ f)(-2)$.

(A) 1 (B) 10 (C) -1 (D) 0 (E) None of these

For the next TWO questions, use the graph of $f(x)$ and the table of values for $g(x)$ below.

x	1	2	3	4	5
$g(x)$	4	-3	5	2	1



The graph of $f(x)$

17. Find $(f \circ g \circ f)(1)$.

(A) -4 (B) 5 (C) 16 (D) -3 (E) None of these

18. Find $(g^{-1} \circ f \circ g)(1)$.

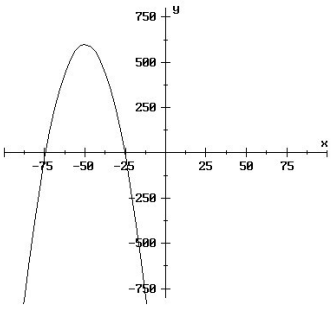
(A) 2 (B) 3 (C) 1 (D) 4 (E) None of these

19. A train that is traveling 52 mph leaves the station and goes west. Another train leaves the station on a parallel track one hour later traveling west at 65 mph. How long will it take the fast train to catch up with the slower train?

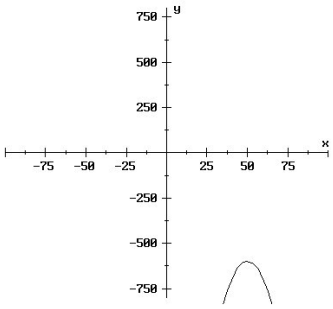
    (A) 5 hours (B) 4 hours (C) 1.25 hours (D) 6 hours (E) None of these

20. Which of the following graphs best represents: $f(x) = 600 - (x - 50)^2$

(A)



(B)



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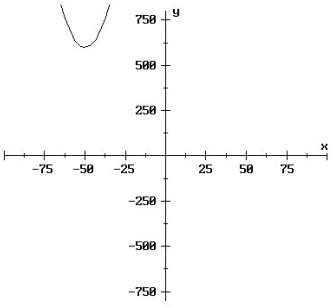

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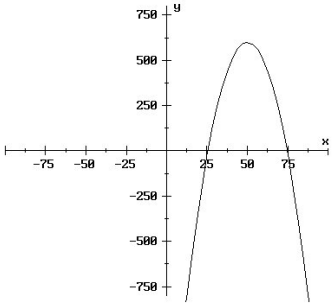
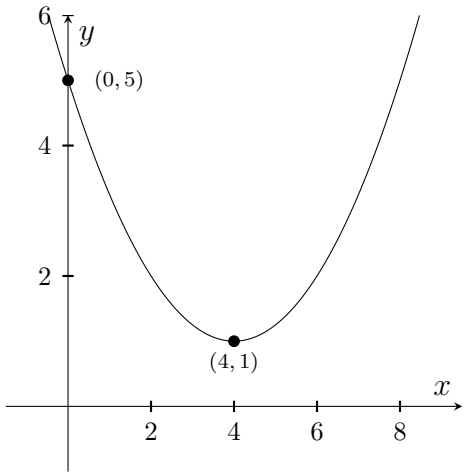


21. Determine a function in standard form for the quadratic function whose graph is shown below.



(A) $f(x) = -\frac{1}{4}(x-1)^2 + 4$

(B) $f(x) = \frac{1}{4}(x+4)^2 + 1$

(C) $f(x) = 4(x-4)^2 + 1$

(D) $f(x) = \frac{1}{4}(x-4)^2 + 1$

(E) $f(x) = -(x-4)^2 + 1$

22. For the function $f(x) = -x^2 + 6x + 62$, which of the following statements is/are true?

(1) the x-coordinate of the vertex is -3
(2) $f(x)$ has a maximum value of 71
(3) one of the x-intercepts is $\left(3 + \sqrt{71}, 0\right)$

(A) 1 and 2 only

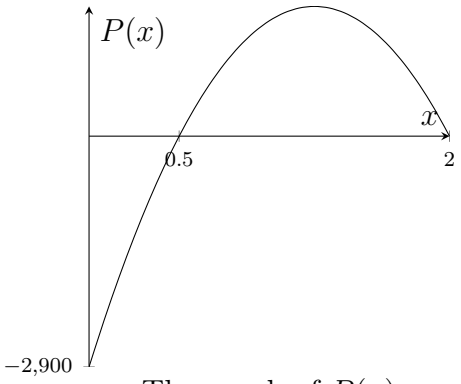
(B) 1 and 3 only

(C) 2 and 3 only

(D) 2 only

(E) 1 only

23. An espresso stand finds that its weekly profit (in dollars), $P(x)$, is a quadratic function of the price, x, it charges per cup (in dollars). A graph of $P(x)$ is shown below:



The graph of $P(x)$

What price should the espresso stand charge to maximize its weekly profits?

(A) \$0.50 per cup (B) \$1.25 per cup (C) \$1.65 per cup (D) \$2.00 per cup

24. Write $f(x) = 3x^2 + 60x - 1$ in standard form. The SUM of the x and y coordinates of the vertex is:

(A) 291 (B) -311 (C) 311 (D) -291 (E) None of these

25. Find the vertex of the quadratic function $y = x^2 + 2ax + b$. The y -coordinate of the vertex is:

(A) b (B) $-b$ (C) $\dfrac{b^2}{4a^2} - b$ (D) $-a^2 + b$ (E) $3a^2 + b$

26. How many x-intercepts does the graph of the function $k(x) = x^2 - 24x + 159$ have?

(A) Zero (B) One (C) Two (D) Three

27. Find the vertex of the quadratic function $f(t) = \frac{4}{7}t^2 - \frac{16}{7}t + 3$.

The y-coordinate of the vertex is:

(A) $\frac{1}{2}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) 1 (E) None of these

28. Find the equation of the parabola that has a vertex of $(3, -1)$ and passes through the point $(4, 1)$. The coefficient of x^2 is a number:

(A) between -2 and -0.5
(B) between -0.5 and 1.5
(C) between 1.5 and 3
(D) between 3 and 4.5
(E) None of these

29. Find the vertex of the quadratic function

$$f(x) = 3x^2 + 7x + 4.$$

The vertex is located:

(A) Above the x-axis
(B) Below the x-axis
(C) At the origin
(D) On the x-axis but not at the origin
(E) None of these

30. A skydiver jumps out of a plane from a height of 13000 feet. Her height (in feet) above the ground t seconds after she jumps is given by the function

$$h(t) = -16t^2 - 24t + 13000.$$

If she plans to open her parachute at a height of 3000 feet, how long after she jumps should she open her parachute? Round your answer to one decimal place.

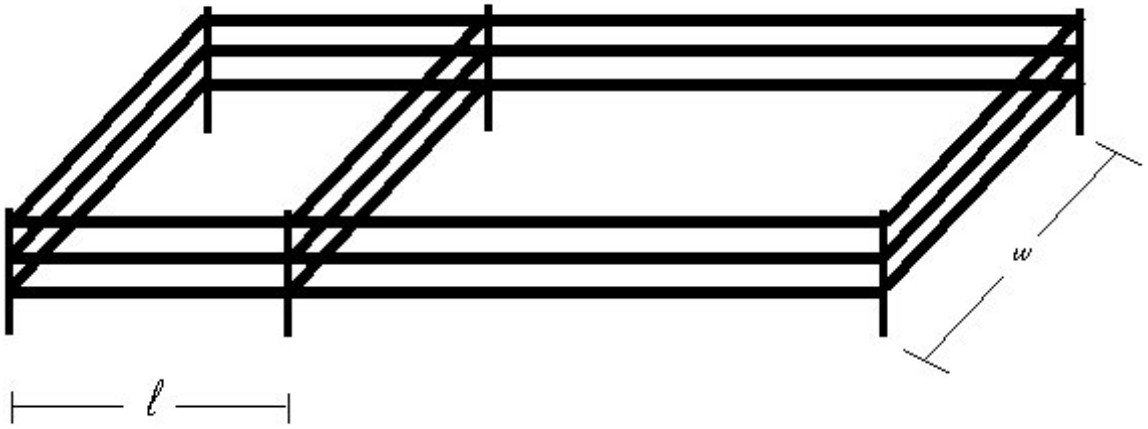
(A) 25.8 seconds
(B) 27.8 seconds
(C) 24.3 seconds
(D) 30.9 seconds
(E) 48.6 seconds

31. A horticulturist has determined that the number of inches a young oak tree grows in one year is a function of the annual rainfall, r, given by $g(r) = -0.01r^2 + 0.1r + 2$. What is the maximum number of inches a young oak can grow in a year? The maximum number of inches is:

(A) less than 1 (B) between 1 and 2 (C) between 2 and 3

(D) between 3 and 4 (E) between 4 and 5

32. A rancher wishes to enclose two adjacent rectangular corrals such that the right-hand corral has twice the length of the left hand (see diagram). She has 900 feet of fencing. What is the maximum area she can enclose?



(A) 22,500 sq. ft. (B) 50,625 sq. ft. (C) 62,500 sq. ft.

(D) 67,500 sq. ft. (E) None of these

33. Which of the following has two x-intercepts?

(1) $f(x) = x^2 - 40$

(2) $g(x) = -x^2 + 60x - 903$

(3) $h(x) = -(x + 125)^2 + 1$

(A) 1 only (B) 2 and 3 only (C) 2 only

(D) 1 and 3 only (E) None of these

34. For the years 1975 through 1990, the average price, p (in dollars per million British thermal units), of fuel used to generate electricity in the U.S. can be modeled by the function

$$p(t) = -0.021t^2 + 0.50t - 1.04$$

where t is time in years since 1970. Estimate the maximum average price, p, of fuel used according to this model.

(A) \$11.9 per million Btu (B) \$1.94 per million Btu (C) \$2.1 per billion Btu

(D) \$10.2 per million Btu (E) None of these

35. Find the following sum: $\sum_{k=3}^{5} (4k - 7)$

(A) -1 (B) 18 (C) 25 (D) 27 (E) None of these

36. Find the following sum: $\sum_{k=1}^{4} (kx + 3)$

(A) 11 (B) 22 (C) $4x + 3$ (D) $5x + 6$ (E) $10x + 12$

37. What is the degree and leading coefficient of

$$f(x) = -3x^2 + 6x^4 - 4x^5 + 7?$$

(A) degree is 2, leading coefficient is -3

(B) degree is 11, leading coefficient is -4

(C) degree is 5, leading coefficient is -4

(D) degree is 5, leading coefficient is 4

(E) None of these

38. For the graph of $y = -3x^4 + 37x^3 + 28x^2 + 42$, which of the following is correct?

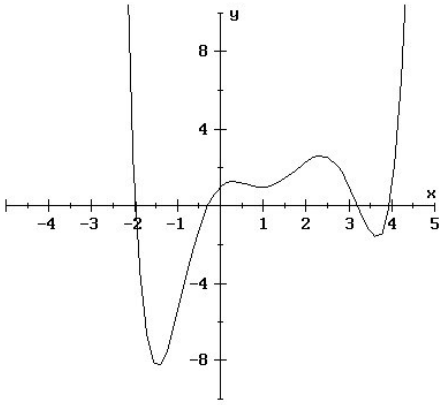
(A) $y \longrightarrow -\infty$ as $x \longrightarrow -\infty$
$y \longrightarrow \infty$ as $x \longrightarrow \infty$

(B) $y \longrightarrow -\infty$ as $x \longrightarrow -\infty$
$y \longrightarrow -\infty$ as $x \longrightarrow \infty$

(C) $y \longrightarrow \infty$ as $x \longrightarrow -\infty$
$y \longrightarrow \infty$ as $x \longrightarrow \infty$

(D) $y \longrightarrow \infty$ as $x \longrightarrow -\infty$
$y \longrightarrow -\infty$ as $x \longrightarrow \infty$

39. Consider the polynomial function below. Which of the following statement(s) is/are true about this polynomial?

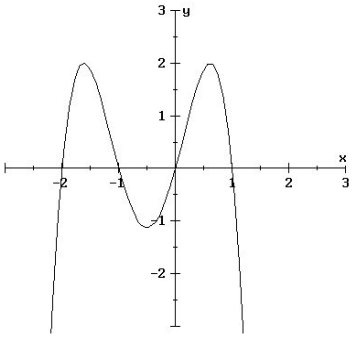


(1) This polynomial could have degree 6
(2) This polynomial could have degree 4
(3) This polynomial could have degree 8

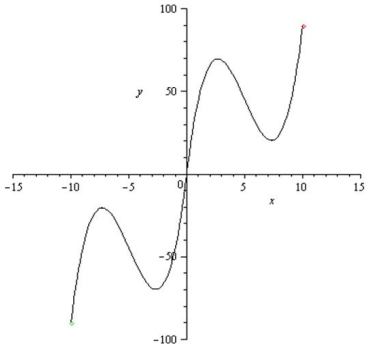
(A) 1 and 2 only (B) 1 and 3 only (C) 1 only
(D) 2 only (E) 2 and 3 only

40. Which of the following could be the graph of a polynomial of odd degree?

(1)



(2)



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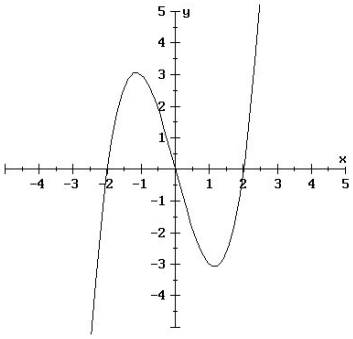

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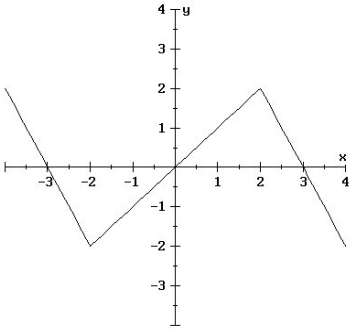


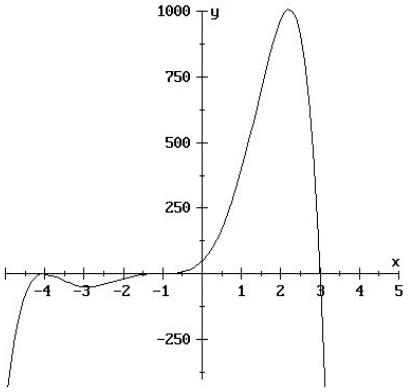
(A) All of them (B) None of them (C) 1, 2 and 3 only
(D) 1, 2 and 4 only (E) 2 and 3 only

41. Which of the following MUST be true?

(1) A polynomial of degree 4 has four unique zeros.
(2) A polynomial of degree 5 has at least 1 real zero.
(3) A polynomial of degree 2 has at least 1 rational zero.

(A) 1 only (B) 2 only (C) 3 only
(D) 1 and 2 only (E) 1 and 3 only

42. Which of the following could be the equation of the polynomial $P(x)$ graphed below?



(A) $P(x) = (x-4)^2(x+3)^2(x-1)$ (B) $P(x) = (x+4)^3(x-3)(x+1)$
(C) $P(x) = -(x+4)^2(x-3)(x+1)^3$ (D) $P(x) = (x-4)^2(x+3)(x+1)$
(E) $P(x) = -(x-4)^2(x+3)(x-1)^3$

43. For the function $f(x) = -3(x-a)^5(x+b)(x-c)^2$, where a, b and c are positive real numbers, which of the following is/are correct?

(1) $f(x)$ is a polynomial of degree 5
(2) $f(x)$ has a zero at $x = b$
(3) $f(x)$ has an x-intercept at $(c, 0)$
(4) $y \longrightarrow -\infty$ as $x \longrightarrow -\infty$ and $y \longrightarrow -\infty$ as $x \longrightarrow \infty$

(A) 1 and 2 only (B) 2 and 4 only (C) 1 and 3 only (D) 3 and 4 only

44. Find all the real zeros of $f(x) = x^3 + 5x^2 + 7x + 2$. The LARGEST real zero is:

(A) $\dfrac{-3+\sqrt{5}}{2}$ (B) -0.5 (C) $\dfrac{-3+\sqrt{13}}{2}$

(D) $\dfrac{-3+\sqrt{7}}{2}$ (E) -2

45. Find all rational zeros of $p(x) = x^4 - 4x^3 + 2x^2 + 5x - 2$. The SUM of all the RATIONAL zeros is:

(A) 1 (B) 2 (C) -1 (D) 0 (E) There are no rational zeros

46. Find the value of 'b' so that $x+2$ is a factor of $P(x) = x^4 - (b+1)x^2 - 5bx - 9b$.

(A) $b = -2$ (B) $b = 4$ (C) $b = 2$ (D) $b = -4$ (E) None of these

47. Find all real zeros of $f(x) = 3x^4 - 36x^2 + 60$. The SMALLEST real zero of $f(x)$ is:

(A) $-\sqrt{2}$ (B) $\frac{7}{5}$ (C) $-\frac{16}{5}$ (D) $-\sqrt{10}$ (E) $\sqrt{2}$

48. Factor $T(x) = x^3 + x^2 + 13x + 30$. One of the factors is:

(A) $x+3$ (B) $x-5$ (C) $x+2$ (D) $x-10$ (E) None of these

49. If 3 is a zero of $f(x) = 18x^3 - 111x^2 + 161x + 30$, what are the other real zeros of $f(x)$? The SMALLEST real zero is:

(A) $-\frac{16}{5}$ (B) $\frac{16}{5}$ (C) $-\frac{1}{6}$ (D) $-\frac{1}{5}$ (E) 0

50. Which of the following statements is/are equivalent to:
"$x+3$ is a factor of the polynomial $f(x)$"?

(1) $x = 3$ is a solution of $f(x) = 0$
(2) $x = -3$ is a zero of $f(x)$
(3) $(-3, 0)$ is an x-intercept of $f(x)$

(A) 1 only (B) 2 only (C) 3 only
(D) 1 and 3 only (E) 2 and 3 only

51. What is the remainder when $p(x) = x^4 + x^3 - x^2 - 2$ is divided by $x+3$?

(A) -26 (B) 43 (C) -17 (D) 0 (E) None of these

52. Find a polynomial of lowest degree that has $-3, -\sqrt{2}$, and $\sqrt{2}$ as its zeros.

(A) $x^3 + 3x^2 - 2x - 6$ (B) $x^2 + (-3 - \sqrt{2})x - 6$
(C) $x^3 + 3x^2 - 4x - 12$ (D) $x^2 - 2\sqrt{2x} + 3$
(E) None of these

53. What is the remainder when $5x^3 - 6x^2 + 3$ is divided by $x^2 - x + 4$?

(A) $-21x + 7$ (B) 7 (C) $21x - 7$
(D) $-14x$ (E) None of these

54. For which of the following functions does $y \longrightarrow \infty$ as $x \longrightarrow -\infty$?

(1) $y = 4x^3 - 3x$
(2) $y = 2x^4 - x + 10$
(3) $y = -x^6 + 3x^5 - 2$

(A) None of them (B) 2 only (C) 2 and 3 only
(D) 1 and 3 only (E) 3 only

55. Find a polynomial of lowest degree having zeros $-2, 1, 0$ (a zero of multiplicity 2), and -4 (a zero of multiplicity 3).

(A) $f(x) = x^2(x + 2)(x - 1)(x + 4)^3$ (B) $f(x) = x^2(x - 2)(x + 1)(x - 4)^3$
(C) $f(x) = 2x(x + 2)(x - 1)(x + 4)^3$ (D) $f(x) = (x + 2)(x - 1)(x + 4)^3$
(E) None of these

56. Find the vertical asymptote(s), if any, for $f(x) = \dfrac{x + 1}{4x^2 - 1}$.

(A) $x = \dfrac{1}{4}$ (B) $x = \dfrac{1}{2}$ (C) $x = -\dfrac{1}{2}, x = \dfrac{1}{2}$

(D) $x = -\dfrac{1}{4}, x = \dfrac{1}{4}$ (E) $x = -1$

57. Which ONE of the following statements is true about the rational function $R(x) = \dfrac{(2x-1)(x-3)(x+1)}{(x+2)(x-3)}$?

(A) $R(x)$ has a horizontal asymptote

(B) $R(x)$ has a slant asymptote

(C) $R(x)$ has two vertical asymptotes

(D) $R(x)$ has three zeros

(E) The domain of $R(x)$ is $(-\infty, -2) \cup (-2, \infty)$

58. Find the value of the parameter a so that the rational function $g(x) = \dfrac{ax^2 + 3x + 7}{x + 1}$ has the slant asymptote $y = 2x + 1$.

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

59. Which of the following statement(s) is/are true about the function $f(x) = \dfrac{6x + 1 + 0.05x^2}{x - 2}$?

(1) $y \longrightarrow 6$ as $x \longrightarrow \infty$

(2) $f(x)$ has exactly one x-intercept

(3) $f(x)$ has a slant asymptote

(A) 1 and 2 only (B) 2 and 3 only (C) 2 only (D) 3 only (E) 1 only

60. Which ONE of the following statements is true about the rational function $g(x) = \dfrac{3x^2 - x}{3x^2 + x}$?

(A) The graph of g has a zero at $x = 0$.

(B) The graph of g has a vertical asymptote at $x = 0$.

(C) The graph of g has a hole at $x = 0$.

(D) The graph of g has a horizontal asymptote at $y = 0$.

(E) The graph of g has no horizontal asymptote.

61. Find all the asymptotes of the rational function $y = \dfrac{2x^2+1}{2x+3}$

(A) $x = -\dfrac{3}{2}, \quad y = x - \dfrac{3}{2}$ (B) $x = -\dfrac{3}{2}, \quad y = 1$ (C) $x = \dfrac{3}{2}, \quad y = x + 3$

(D) $x = \dfrac{3}{2}, \quad y = 1$ (E) $x = -\dfrac{3}{2}, \quad y = x + \dfrac{1}{2}$

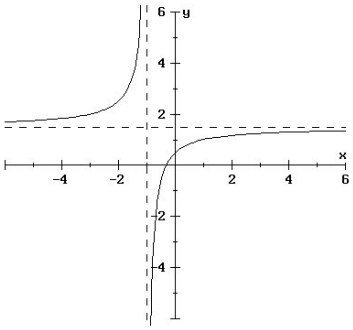
62. Which of the following rational functions has no vertical asymptote?

(1) $y = \dfrac{2}{x^2-5}$ (2) $y = \dfrac{5x-4}{x^2+1}$ (3) $y = \dfrac{7x^2}{-x^2+3}$

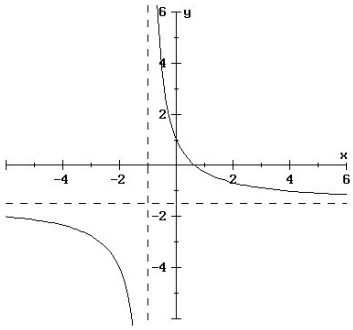
(A) 1 and 3 only (B) 1 and 2 only (C) 2 and 3 only
(D) 2 only (E) 1 only

63. Which of the following is the graph of $y = \dfrac{2x+1}{2x-3}$

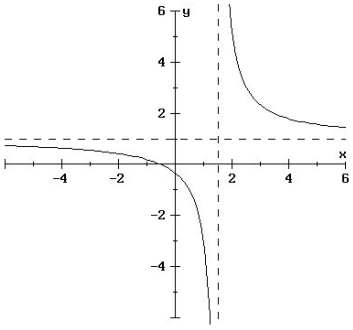
(A)



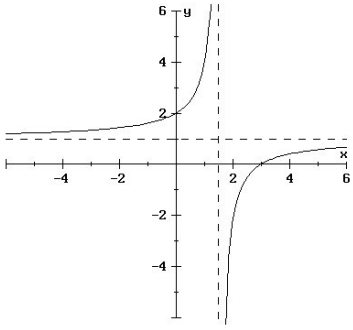
(B)



(C)



(D)



(E) None of these

64. Which ONE of the following rational functions has a graph that:

(i) Has vertical asymptotes at $x = 20$ and $x = -20$
(ii) Has an x-intercept $(1, 0)$
(iii) Has a horizontal asymptote at $y = 0$

(A) $f(x) = \dfrac{x^2 - 400}{x + 1}$ (B) $f(x) = \dfrac{x + 1}{x^2 - 20}$ (C) $f(x) = \dfrac{x - 1}{x^2 - 400}$

(D) $f(x) = \dfrac{x + 1}{x^2 - 20}$ (E) None of these

65. Determine the behavior of the function $f(x) = \dfrac{2x + 1}{1 - 18x}$ as $x \longrightarrow \infty$.

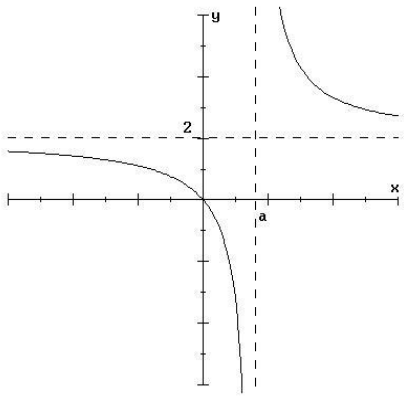
(A) $y \longrightarrow -\dfrac{1}{9}$ (B) $y \longrightarrow 2$ (C) $y \longrightarrow 0$ (D) $y \longrightarrow \infty$
(E) $y \longrightarrow \dfrac{1}{18}$

66. Which of the following is/are TRUE about the function $R(x) = \dfrac{15}{x^2 - x - 6}$

(1) $x = 15$ is a zero of $R(x)$.
(2) $R(x)$ has a y-intercept of $\left(0, -\frac{5}{2}\right)$.
(3) $R(x)$ has no real zeros.

(A) 1 only (B) 2 only (C) 3 only
(D) 2 and 3 only (E) 1 and 2 only

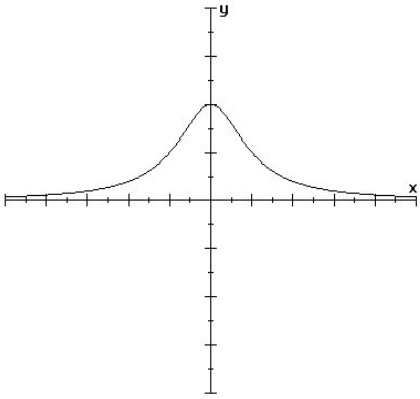
67. Find a formula for the rational function show below.



(A) $y = \dfrac{2}{x-a}$ (B) $y = \dfrac{2x}{x+a}$ (C) $y = \dfrac{2x}{x-a}$

(D) $y = \dfrac{x+2}{x-a}$ (E) $y = \dfrac{x-a}{2x}$

68. If $y = \dfrac{1}{f(x)}$ is the equation for the graph below, which of the following are NOT possible expressions for $f(x)$?



(1) $x-4$ (2) x^2+1 (3) x^2-4 (4) x

(A) 2 only (B) 1 and 4 only (C) 2 and 3 only
(D) 1, 3 and 4 only (E) All are possible

69. The graph of a particular rational function has one vertical asymptote at $x = 2$, a horizontal asymptote at $y = 1$, and passes through the point $(4, 5)$. What is the x-intercept?

(A) $(-6, 0)$ (B) $(-7, 0)$ (C) $(2, 0)$ (D) $(1, 0)$

70. Which of the following is/are correct for the function $f(x) = a^{-x} (a > 1)$?

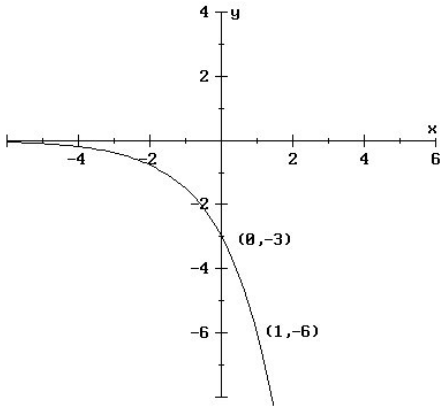
(1) $f(x)$ is decreasing (2) The domain is $(-\infty, 0]$
(3) The range is $(0, \infty)$ (4) The y-intercept is $(0, 1)$

(A) 1 and 4 only (B) 3 and 4 only (C) 1 only
(D) 1, 3 and 4 only (E) 2 only

71. Find the domain and range of the function $H(x) = 3^{x+2} - 4$.

(A) Domain: $(-2, \infty)$ Range: $(-4, \infty)$
(B) Domain: $(-\infty, \infty)$ Range: $(-4, \infty)$
(C) Domain: $(-\infty, \infty)$ Range: $(0, \infty)$
(D) Domain: $(2, \infty)$ Range: $(4, \infty)$
(E) Domain: $(-2, \infty)$ Range: $(4, \infty)$

72. The graph below represents $y = C(a)^x$. Find the values of C and a.



(A) $C = -3, a = 2$ (B) $C = -3, a = 3$ (C) $C = 2, a = -2$
(D) $C = 2, a = -3$ (E) $C = \frac{1}{3}, a = -2$

73. How much MORE money will you earn in an account that compounds interest continually than in an account that compounds interest quarterly if you invest \$3000 for 7 years at an interest rate of 11%?

(A) \$67.02 (B) \$59.37 (C) \$101.16 (D) \$32.52 (E) None of these

74. For the natural log function $y = \ln(x)$, which of the following is/are correct?

(1) The graph is increasing (2) The x-intercept is $(1, 0)$
(3) The graph is continuous (4) The y-intercept is $(0, e)$

(A) All of them (B) 1, 2 and 3 only (C) 2, 3 and 4 only
(D) 2 and 3 only (E) 1 and 2 only

75. Find the x-intercept of the graph of $y = \ln(x - a) + 2$

(A) $(\ln(-a) + 2, 0)$ (B) $(-2 + \ln a, 0)$ (C) $(e^{-2+\ln(a)}, 0)$
(D) $(e^{-2} + a, 0)$ (E) None of these

76. The given table lists the day, y, that there were x bacteria (in thousands).

x	1	10	100	1,000	10,000
y	1	3	5	7	9

Find the values of a and b so that $f(x) = a + b\log(x)$ models the data exactly.

(A) $a = 1,\ \ b = 2$ (B) $a = 2,\ \ b = 2$ (C) $a = 1,\ \ b = 1$
(D) $a = 2,\ \ b = 1$ (E) None of these

77. If $f(x)$ is a one-to-one function, and $f(2) = 7$, then which of the following CANNOT be true?

(A) $f(7) = 2$ (B) $f^{-1}(7) = 2$ (C) $f^{-1}(5) = 3$
(D) $f(-2) = 4$ (E) $f(-2) = 7$

78. Consider the functions $f(x)$ and $g(x)$ represented by the tables shown below.

x	$f(x)$
2	4
3	3
4	6
5	5

x	$g(x)$
2	4
3	5
4	3
5	4

Which, if either, of these functions is one-to-one?

(A) both $f(x)$and $g(x)$ (B) $g(x)$only (C) $f(x)$only
(D) neither $f(x)$nor $g(x)$

79. If $h(x) = 20x - 62$, what is $h(h^{-1}(6))$?

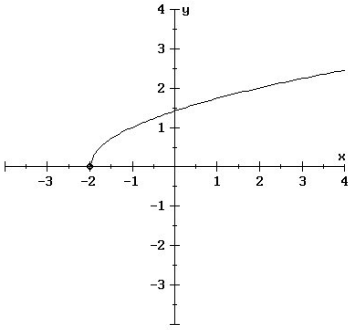
(A) 2 (B) 58 (C) -6 (D) 6 (E) None of these

80. Which of the following functions is/are one-to-one?

(1) $f(x) = \dfrac{6}{x - 13}$ (2) $g(x) = 0.05(x + 3)$ (3) $h(x) = \sqrt{3x^2 - 40}$

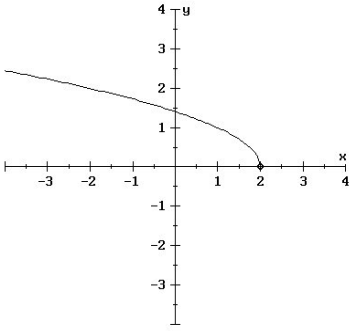
(A) 2 and 3 only (B) 1 and 2 only (C) 1, 2 and 3 only
(D) 1 and 3 only (E) None are one-to-one

81. If the graph of $S(x)$ is:

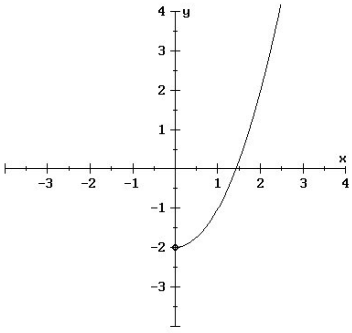


Then the graph of $S^{-1}(x)$ is:

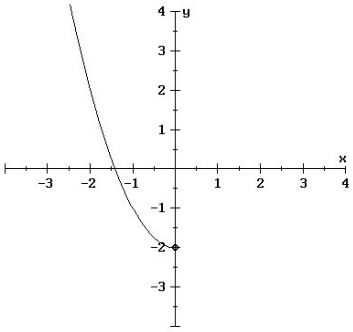
(A)



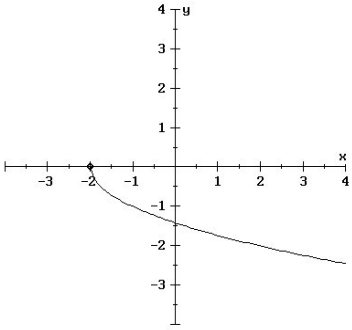
(B)



(C)



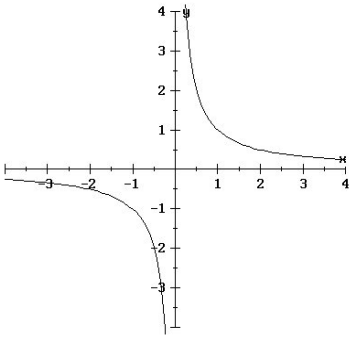
(D)



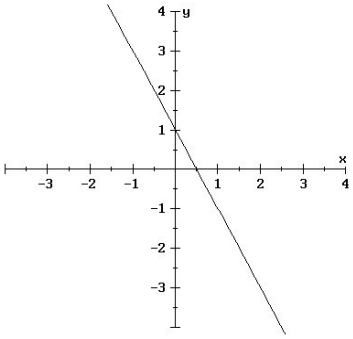
(E) None of these

82. Of the functions graphed below, which have inverse functions?

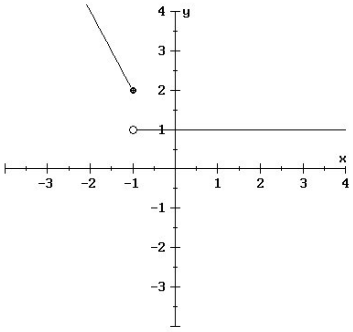
(1)



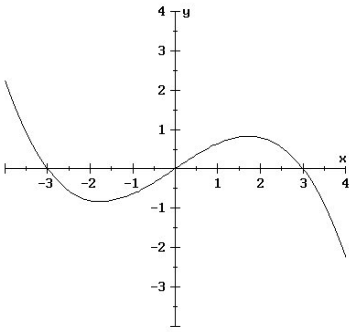
(2)



(3)



(4)



(A) 1, 2 and 3 only (B) 2 only (C) 1 and 2 only
(D) 1, 2 and 4 only (E) None these

83. If $G(x) = \sqrt[3]{4-x}$, what is $G^{-1}(x)$?

(A) $G^{-1}(x) = \dfrac{1}{\sqrt[3]{4-x}}$ (B) $G^{-1}(x) = -\sqrt[3]{4-x}$ (C) $G^{-1}(x) = 4 - x^3$
(D) $G^{-1}(x) = x^3 + 4$ (E) None of these

84. If the point $(2, 6)$ is on the graph of $f(x)$, which of the following points must be on the graph of $f^{-1}(x)$?

(A) $(-2, -6)$ (B) $\left(2, \dfrac{1}{6}\right)$ (C) $(6, 2)$ (D) $(2, -6)$ (E) None of these

85. Find $R^{-1}(x)$ if $R(x) = \dfrac{C}{3x-1}$ (C is a real number)

(A) $R^{-1}(x) = \dfrac{C}{3}x + C$ (B) $R^{-1}(x) = \dfrac{3x-1}{C}$

(C) $R^{-1}(x) = \dfrac{C+x}{3x}, x \neq 0$ (D) $R^{-1}(x) = \dfrac{C-3x}{x}, x \neq 0$

(E) None of these

86. Which of the following is/are correct? $[a > 0, a \neq 1]$

(1) $\log_a 1 = 0$ (2) $\log_a 0 = 0$ (3) $\ln 1 = e$ (4) $\ln e^a = a$

(A) None of them
(B) 4 only
(C) 1 and 4 only
(D) 3 only
(E) 2 and 4 only

87. Suppose $g(x)$ is the inverse of the function $f(x)$. Which of the following tables would be correct for $g(2x)$ if $f(x)$ is given in the table below.

x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

(A)

x	1	3	5	7	9
$g(2x)$	-4	-2	0	2	4

(B)

x	1	3	5	7	9
$g(2x)$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1

(C)

x	2	6	10	14	18
$g(2x)$	-2	-1	0	1	2

(D)

x	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
$g(2x)$	-2	-1	0	1	2

88. Let $f(x) = \dfrac{e^x + 1}{e^x - 1}$. Find $f^{-1}(x)$.

(A) $f^{-1}(x) = \ln\left(\dfrac{e^x + 1}{e^x - 1}\right)$ (B) $f^{-1}(x) = \dfrac{\ln(e^x + 1)}{\ln(e^x - 1)}$ (C) $f^{-1}(x) = \ln\left(\dfrac{x + 1}{x - 1}\right)$

(D) $f^{-1}(x) = \dfrac{\ln(x + 1)}{\ln(x - 1)}$ (E) $f^{-1}(x) = \dfrac{\ln(x - 1)}{\ln(x + 1)}$

89. Let f be a function with values given in the table below.

x	0	1/5	2	3	4	5
$f(x)$	4	1	36	25	9	16

Find $h^{-1}\left(\frac{1}{5}\right)$ for $h(x) = \dfrac{1}{\sqrt{f(x)}}$.

(A) 25 (B) 16 (C) 3 (D) 1 (E) 1/4

90. Let f be a function with values given in the table below.

x	$f(x)$
0	2
1	-1
2	1
3	5
4	3
5	4

Which of the following functions is/are invertible?

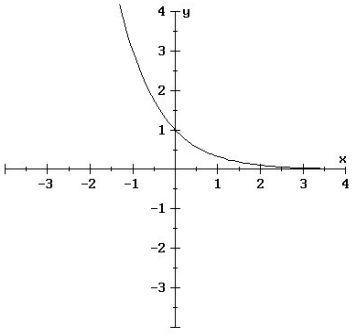
$h(x) = (f(x))^2$ $g(x) = \dfrac{1}{f(x)}$ $p(x) = \dfrac{1}{\sqrt{f(x) + 2}}$

(A) g only (B) p only (C) h and g only

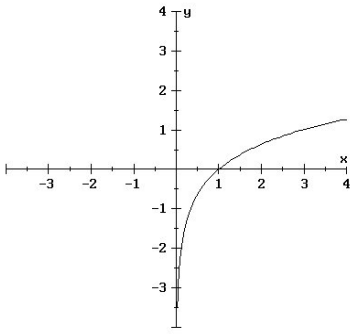
(D) g and p only (E) h and p only

91. Which of the following is a graph of the INVERSE of $y = \log_3 x$?

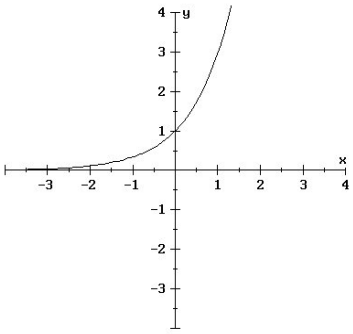
(A)



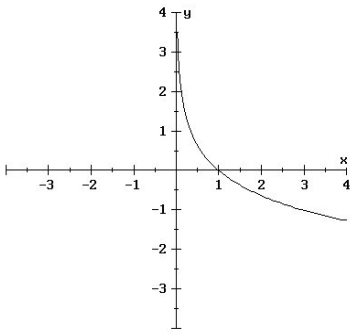
(B)



(C)



(D)



(E) None of these

92. For the function $R(x) = \log_2 x$, which of the following is/are correct?

(1) The domain is $[0, \infty)$ (2) The range is $(-\infty, \infty)$ (3) $R(x)$ is one-to-one

(A) 2 and 3 only (B) 1 and 2 only (C) 3 only
(D) 1 and 3 only (E) All of them

93. What is the domain of $y = 10 - \log_7(5n - 230)$?

(A) $(-\infty, 230]$ (B) $[46, \infty)$ (C) $(46, \infty)$
(D) $(230, \infty)$ (E) $(-\infty, 48)$

94. Find the x-intercept of the graph of $M(x) = \log_6(2x + 3)$.

(A) $(1.5, 0)$ (B) $(0, 0)$ (C) $(-3, 0)$
(D) $(0.5, 0)$ (E) $(-1, 0)$

95. Rewrite $23^b = a$ in logarithmic form.

(A) $\log_a b = 23$ (B) $\log_{23} a = b$ (C) $\log_{23} b = a$
(D) $\log_b 23 = a$ (E) None of these

96. Find the exact value of $\ln\left[\sqrt[4]{e^5}\right]$.

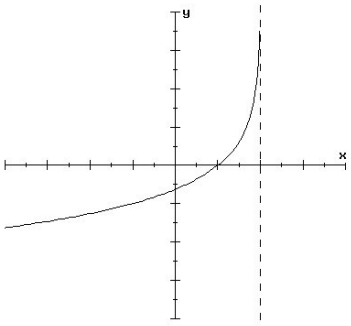
(A) 0.8 (B) 1.25 (C) e (D) $0.8e$ (E) None of these

97. Solve $\log_5 x = 2$ and $\log_2 32 = w$. The two solutions are:

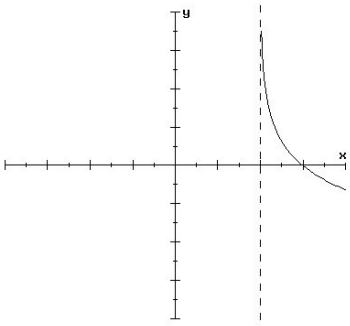
(A) $x = \sqrt{5}$ and $w = \sqrt{32}$ (B) $x = 25$ and $w = 5$ (C) $x = 32$ and $w = 16$
(D) $x = 25$ and $w = 16$ (E) None of these

98. Which of the following most resembles the graph of $y = -\log_3(x - a)$, where a is a positive constant?

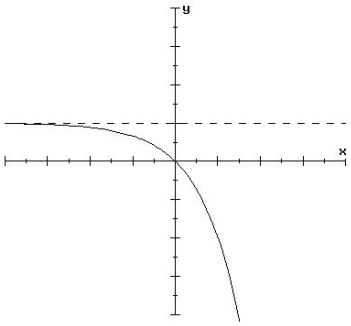
(A)



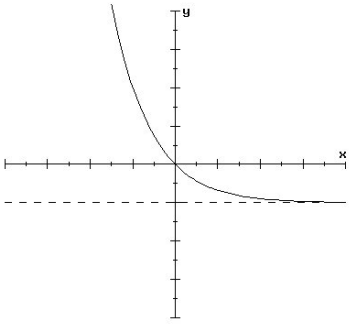
(B)



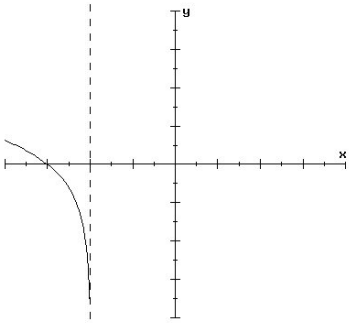
(C)



(D)



(E)



99. If M and N are positive, which of the following is/are correct?

(1) $\ln(MN) = \ln M + \ln N$ (2) $\ln(M+N) = \ln M + \ln N$

(3) $\dfrac{\ln M}{\ln N} = \ln M - \ln N$

(A) All of them (B) None of them (C) 1 only
(D) 1 and 3 only (E) 2 and 3 only

100. Express as a single logarithm and simplify if possible:

$$\frac{1}{3}\log_a x + 4\log_a y - 2\log_a z$$

(A) $\log_a[x^{1/3} + y^4 - z^2]$ (B) $\log_a\left[\dfrac{1}{3}x + 4y - 2z\right]$ (C) $\dfrac{7}{3}\log_a\left[\dfrac{xy}{z}\right]$
(D) $\log_a\left[\dfrac{x^{1/3}y^4}{z^2}\right]$ (E) None of these

101. Express as a sum or difference of the natural logarithms of x, y, and z: $\ln\sqrt{\dfrac{x^3y}{z^4}}$

(A) $\dfrac{(3\ln x)(\ln y)}{4\ln z}$ (B) $\dfrac{3}{2}\ln x + \ln y - 4\ln z$
(C) $\dfrac{3}{2}\ln x + \dfrac{1}{2}\ln y - 2\ln z$ (D) $6\ln x + 2\ln y - 4\ln z$
(E) None of these

102. Use natural logarithms to solve for x : $3 + 6e^{2x} = 5$

(A) $x = \dfrac{1}{2}\ln 3$ (B) $x = 2\ln 3$ (C) $x = \dfrac{1}{2}\ln\left(\dfrac{1}{3}\right)$
(D) $x = \ln 3 - 2$ (E) None of these

103. Determine the domain of the logarithmic function $f(x) = \log_7(x-4)$.

(A) $(-\infty, 4)$ (B) $(-\infty, 4]$ (C) $(-\infty, 1)$ (D) $[4, \infty)$ (E) $(4, \infty)$

104. Solve for x : $3^{2x} = 27^{2x-1}$

The answer is a number:

(A) between 0.5 and 1 (B) between 0 and 0.5 (C) between -0.5 and 0
(D) between 1 and 2 (E) None of these

105. Solve for x : $3^x = 5^{x-1}$

The solution is a number:

(A) between 2 and 4 (B) between -5 and -3 (C) between -1 and 0
(D) between -3 and -1 (E) None of these

106. Solve for x : $\ln(2x-1) = 2$

(A) $x = \dfrac{e^2}{2}$ (B) $x = \dfrac{e^2+1}{2}$ (C) $x = e^2 + \dfrac{1}{2}$ (D) $x = \dfrac{e^4}{2}$
(E) None of these

107. Solve for x: $\log_2(-4-x) + \log_2(3-x) = 3$

The solution is a number:

(A) between -9 and -6 (B) between -6 and -4
(C) between -4 and -1 (D) between -1 and 2
(E) between 2 and 5

108. Solve for x : $\log_3 x - \log_3(x-1) = 2$

(A) $\frac{1}{8}$ (B) $\frac{9}{8}$ (C) $\frac{10}{9}$ (D) $\frac{100}{99}$ (E) None of these

109. The number of California gray whales is growing according to the formula $A = Pe^{0.015t}$, where t is measured in years. How long will it take the number of whales to double?

(A) less than 40 years
(B) between 40 and 50 years
(C) between 50 and 60 years
(D) between 60 and 70 years
(E) more than 70 years

110. In 1980, the population of the United States was approximately 226.5 million people. In 1990, the population had grown to approximately 246.7 million. Assuming an exponential growth model $A = Pe^{rt}$, what is the projected population of the U.S. in the year 2000?

(A) Less than 260 million
(B) Between 260 million and 265 million
(C) Between 265 million and 270 million
(D) Between 270 million and 275 million
(E) More than 275 million

111. Mr. Smart decided to invest \$20,000 in a savings account. At what annual percentage rate, compounded monthly, did he invest his money in order to have \$36,500 at the end of 10 years?

The interest rate is:

(A) less than 6.1%
(B) between 6.1% and 6.3%
(C) between 6.3% and 6.5%
(D) between 6.5% and 6.7%
(E) greater than 6.7%

112. An object which is initially at a temperature of 100°F is placed in a room which is at a constant temperature of 60°F. Given that the rate of change is $r = -0.05$ when t is measured in minutes, determine how long it will take the object to cool to a temperature of 70°F.
Recall that Newton's Law of Cooling is $A(t) = C + (A_0 - C)e^{rt}$.

(A) 61.2 minutes (B) 1.4 minutes (C) 26.3 minutes

(D) 27.7 minutes (E) None of these

113. The release of fluorocarbons used in household sprays destroys the ozone layer in the upper atmosphere. Suppose the amount of ozone is given by $P = Ce^{-0.0025t}$ where t is measured in years. How long will it take for 70% of the ozone to disappear? (Round to the nearest yr.)

(A) About 143 yrs. (B) About 1699 yrs. (C) About 1360 yrs.
(D) About 482 yrs. (E) None of these

Answers

1. B
2. C
3. A
4. D
5. C
6. B
7. D
8. E
9. B
10. A
11. B
12. D
13. C
14. D
15. E
16. C
17. A
18. B
19. B
20. D
21. D
22. C
23. B
24. B
25. D
26. A
27. B
28. C
29. B
30. C
31. C
32. E
33. D
34. B
35. D
36. E
37. C
38. B
39. B
40. E
41. B
42. C
43. D
44. A
45. A
46. B
47. D
48. C
49. C
50. E
51. B
52. A
53. A
54. B
55. A
56. C
57. B
58. B
59. D
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61. A
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69. A
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94. E
95. B
96. B
97. B
98. B
99. C
100. D
101. C
102. C
103. E
104. A
105. A
106. B
107. B
108. B
109. B
110. C
111. A
112. D
113. D