Math 129 - Section 12
Exam #1

Instructors Name: Ryan Smith
Students Name (please print): Solutions____________________________________

By signing my name below, I agree that I am following all rules and regulations set forth by the Code of Academic Integrity. Furthermore, I agree that I am following all rules set by my instructor and by the course policy for this exam.

Signature: ____

Date:____

You should work alone and show all of your work on all the problems. You may not use a calculator.

1. Use any method except an integral table to find the following integrals:

   (a) $\int x^3 \cos(x^4) dx$

   Use the substitution $u = x^4$, $du = 4x^3 dx$

$$\begin{aligned}\int x^3 \cos(x^4) dx &= \int \frac{1}{4} \cos(u) du = \frac{1}{4} \sin(u) + C \\ &= \frac{1}{4} \sin(x^4) + C\end{aligned}$$

   (b) $\int \frac{1}{z \ln(z)} dz$

   Use the substitution $u = \ln(z)$, then $du = \frac{dz}{z}$

$$\int \frac{1}{z \ln(z)} dz = \int \frac{1}{u} du = \ln|u| + C = \ln|\ln(z)| + C$$

   (c) $\int \arctan(x) dx$

   Integration by parts with $u = \arctan(x)$ and $dv = dx$. Then $du = \frac{1}{x^2+1} dx$, $v = x$.

$$\int \arctan(x) dx = x \arctan(x) - \int \frac{x}{x^2 + 1} dx$$

   Substitute $u = x^2 + 1$, $du = 2x dx$.

$$\begin{aligned}&= x \arctan(x) - \int \frac{1}{2} \frac{1}{u} du = x \arctan(x) - \frac{1}{2} \ln|u| + C \\ &= x \arctan(x) - \frac{1}{2} \ln|x^2 + 1| + C\end{aligned}$$

2. Consider the following integral $\int \frac{x+1}{x^2+4x+4}dx$

(a) Use partial fractions to find the values of A and B for which

$$\frac{x+1}{x^2+4x+4} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

$$\begin{aligned}\frac{x+1}{x^2+4x+4} &= \frac{A}{x+2} + \frac{B}{(x+2)^2} \\ x+1 &= A(x+2)+B \\ x+1 &= Ax+(2A+B)\end{aligned}$$

which forces $A = 1$,

$$\begin{aligned}1 &= 2(1)+B \\ B &= -1\end{aligned}$$

$A =$ 1 $B =$ -1

(b) Use your results from the previous part to evaluate $\int \frac{x+1}{x^2+4x+4}dx$

$$\int \frac{x+1}{x^2+4x+4}dx = \int \left(\frac{1}{x+2} - \frac{1}{(x+2)^2}\right) dx$$

Now use $u = x+2$, $du = dx$ substitution,

$$\begin{aligned}&= \int \left(\frac{1}{u} - \frac{1}{u^2}\right) du = \ln|u| + u^{-1} + C \\ &= \ln|x+2| - \frac{1}{x+2} + C\end{aligned}$$

3. Given that $f(1) = 2$, $f(3) = -5$, $\int_1^3 f(x)dx = 5$, and $\int_1^3 xf(x)dx = 3$, find

(a) $\displaystyle\int_1^3 3f(x)dx$

$$\int_1^3 3f(x)dx = 3\int_1^3 f(x)dx = 3(5) = 15$$

(b) $\displaystyle\int_{-2}^0 f(1-x)dx$

Use the substitution $u = 1 - x$, so $du = -dx$

$$\int_{-2}^0 f(1-x)dx = \int_3^1 f(u)(-du) = \int_1^3 f(u)du = 5$$

(c) $\displaystyle\int_1^3 x^2 f'(x)dx$

Use integration by parts with $u = x^2$, $dv = f'(x)dx$, $du = 2xdx$, $v = f(x)$.

$$\int_1^3 x^2 f'(x)dx = \left[x^2 f(x)\right]_1^3 - \int_1^3 2xf(x)dx = 9f(3) - 1f(1) - 2\int_1^3 xf(x)dx$$
$$= 9(-5) - 1(2) - 2(3) = -53$$

4. What is the appropriate trigonometric substitution to evaluate each integral? You do **not** have to integrate, just state the substitution you would use.

   (a) $\displaystyle\int \frac{t}{\sqrt{9t^2-1}}dt$
   Use the substitution $t = \frac{1}{3}\sec(\theta)$ since $\sec^2(\theta) - 1 = \tan^2(\theta)$

   (b) $\displaystyle\int \frac{1}{(y^2+64)^{11}}dy$
   Use the substitution $y = 8\tan(\theta)$ since $\tan^2(\theta) + 1 = \sec^2(\theta)$

5. State whether each of the following statements is true or false. You do not need to show any work.

   (a) If $g(x)$ is never zero, then $\displaystyle\int \frac{1}{g(x)}dx = \ln|g(x)| + C$.
   False, the statement is missing $g'(x)$ in the numerator.

   (b) Using the substitution $x = \sqrt{5}\sec\theta$, the integral $\displaystyle\int \frac{\sqrt{x^2-5}}{x}dx$ is equal to $\displaystyle\int \frac{\tan\theta}{\sec\theta}d\theta$.
   False, this is what the substitution would look like if you forgot to substitute for dx.

   (c) If $f(x)$ is positive, continuous and decreasing on the interval $[a, b]$ then we must have $\displaystyle\int_a^b f(x)dx < LEFT(n)$.
   True. It is decreasing, so the value at the left edge of each interval is greater than the value on the rest of the interval, so the left approximation over-estimates.

6. **For this problem only**, you may use the fact that

$$\int \frac{1}{(x-a)(x-b)} dx = \frac{1}{b-a} \left(\ln|x-a| - \ln|x-b| \right) + C$$

Use this to find $\displaystyle\int \frac{x^4 - x^3 - 4x^2 - 5x - 2}{x^2 - 2x - 3} dx$

The first step is to do polynomial long division so that we can apply partial fractions, when we do this we get

$$\int \frac{x^4 - x^3 - 4x^2 - 5x - 2}{x^2 - 2x - 3} dx = \int \left(x^2 + x + 1 + \frac{1}{(x-3)(x+1)} \right) dx$$

Then we take $a = -1$, $b = 3$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x + \frac{1}{4} \left(\ln|x+1| - \ln|x-3| \right) + C$$