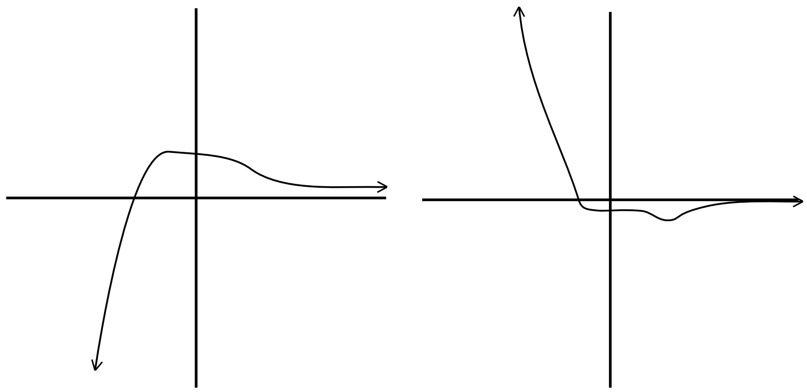
Math 124 - Section 2
Exam #2

Instructors Name: Ryan Smith
Students Name (please print): Solutions

1. The graph of the function $y = f(x)$ is shown below, sketch the graph of $\frac{dy}{dx}$



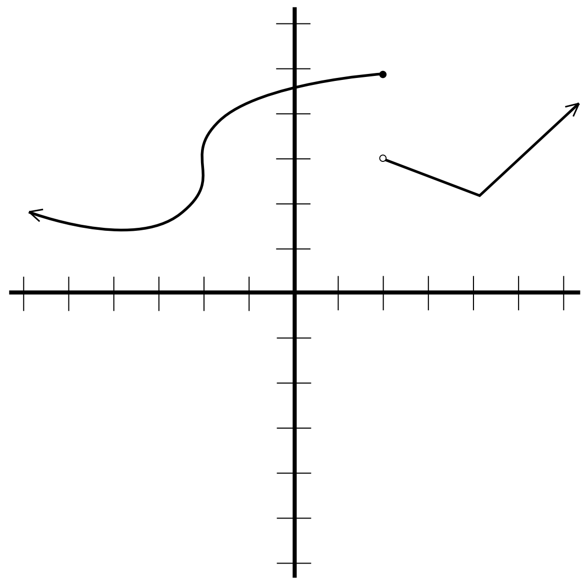
2. Compute $\frac{d}{dx}\left(\pi^x + x^\pi + 4\right)$

$$\frac{d}{dx}\left(\pi^x + x^\pi + 4\right) = (\ln \pi)\pi^x + \pi x^{\pi - 1}$$

3. Find $f'(t)$ where $f(t) = \frac{e^t - 1}{t^2 + 1}$

$$\frac{d}{dt}\left(\frac{e^t - 1}{t^2 + 1}\right) = \frac{(t^2+1)\frac{d}{dt}\left(e^t - 1\right) - (e^t - 1)\frac{d}{dt}\left(t^2+1\right)}{(t^2+1)^2} = \frac{(t^2+1)e^t - (e^t - 1)(2t)}{(t^2+1)^2}$$

4. Let $\Lambda(p)$ be the function shown below.



(a) On what intervals is $\Lambda(p)$ continuous? $(-\infty, 2) \cup (2, \infty)$

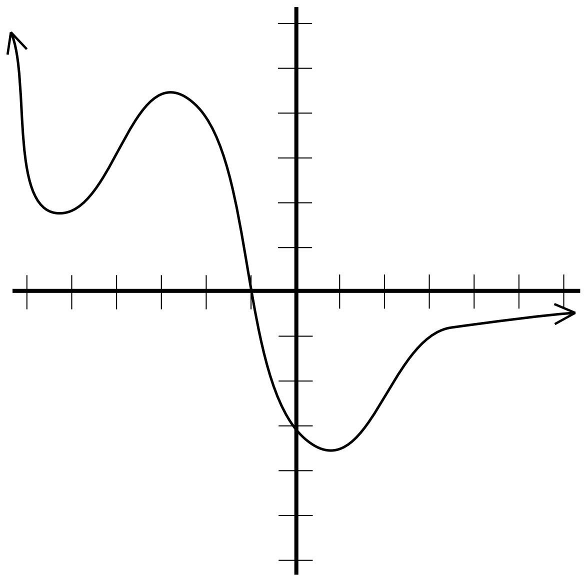
(b) On what intervals is $\Lambda(p)$ differentiable? $(-\infty, -2) \cup (-2, 2) \cup (2, 4) \cup (4, \infty)$

5. Let $h(t) = \frac{2}{3}t^3 - 4t^2 - 8t + \frac{2}{3}$. For which values of t is the tangent line of $h(t)$ parallel to the line $y = 2x - 5$.

`First note that the slope of` $y = 2x - 5$ `is always 2, so the problem is asking when` $h'(t) = 2$. $h'(t) = 2t^2 - 8t - 8$`, so all that remains is solving the quadratic:`

$$\begin{aligned} 2t^2 - 8t - 8 &= 2 \\ t^2 - 4 - 5 &= 0 \\ t &= -1, 5 \end{aligned}$$

6. The graph of the function $M(z)$ is shown below.



(a) For what z does $M(z) = 0$? $z = -1$

(b) For what z does $M'(z) = 0$? $z = -5, -3, 1$

(c) For what z does $M''(z) = 0$? $z = -4, -1, 2$

7. Find the derivative of $J(\theta) = \sin(\theta^2)\sec(2\theta)$

`We do not need to remember the formula for the derivative of secant, it it enough to know that secant is the reciprocal of cosine.`

$$\begin{aligned}\frac{d}{d\theta}\left(\sin(\theta^2)\sec(2\theta)\right) &= \frac{d}{d\theta}\left(\sin(\theta^2)(\cos(2\theta))^{-1}\right)\\ &= \cos(\theta^2)(2\theta)(\cos(2\theta))^{-1} + \sin(\theta^2)(-1)(\cos(2\theta))^{-2}(-\sin(2\theta))(2)\\ &= 2\theta\cos(\theta^2)\sec(2\theta) + 2\sin(\theta^2)\sin(2\theta)\sec^2(2\theta)\end{aligned}$$

8. Find $\frac{d}{dz}\left(\sqrt[4]{z^2+4e^z}\right)$

$$\frac{d}{dz}\left(\sqrt[4]{z^2+4e^z}\right) = \frac{1}{4}(z^2+4e^z)^{-\frac{3}{4}}\frac{d}{dz}\left(z^2+4e^z\right) = \frac{1}{4}(z^2+4e^z)^{-\frac{3}{4}}(2z+4e^z)$$
$$= \frac{z+2e^z}{2(z^2+4e^z)^{\frac{3}{4}}}$$

9. Determine $\frac{d}{dt}\left(\ln\left(\arctan t\right)\right)$

$$\frac{d}{dt}\left(\ln\left(\arctan t\right)\right) = \frac{1}{\arctan(t)}\frac{d}{dt}\left(\arctan(t)\right) = \left(\frac{1}{\arctan(t)}\right)\left(\frac{1}{1+t^2}\right)$$
$$= \frac{1}{(1+t^2)\arctan(t)}$$

10. Let $f(x)$ and $g(x)$ be functions whose values are given in the following tables. Let $h(x) = f(g(x))$, find $h'(1)$. (Show work, otherwise it is all or nothing).

x	-2	-1	0	1	2	3	4
$f(x)$	6	3	-1	2	4	11	45

x	-2	-1	0	1	2	3	4
$f'(x)$	-3	-4	$\frac{1}{2}$	2	3	7	14

x	-2	-1	0	1	2	3	4
$g(x)$	-8	-2	0	3	-5	-11	-20

x	-2	-1	0	1	2	3	4
$g'(x)$	5	1	0	-2	-3	-5	-8

`By the chain rule we have` $h'(x) = f'(g(x))g'(x)$`, so in particular`

$$h'(1) = f'(g(1))g'(1) = f'(3)(-2) = 7(-2) = -14$$

11. Find $\frac{d}{ds}\left(\sqrt{\frac{1+s^2}{1-3^s}}\right)$

$$\begin{aligned}
\frac{d}{ds}\left(\sqrt{\frac{1+s^2}{1-3^s}}\right) &= \frac{1}{2}\left(\frac{1+s^2}{1-3^s}\right)^{-\frac{1}{2}}\frac{d}{ds}\left(\frac{1+s^2}{1-3^s}\right) \\
&= \frac{1}{2}\left(\frac{1+s^2}{1-3^s}\right)^{-\frac{1}{2}}\left(\frac{(1-3^s)\frac{d}{ds}(1+s^2)-(1+s^2)\frac{d}{ds}(1-3^s)}{(1-3^s)^2}\right) \\
&= \frac{1}{2}\left(\frac{1+s^2}{1-3^s}\right)^{-\frac{1}{2}}\left(\frac{(1-3^s)2s-(1+s^2)(-\ln 3)3^s}{(1-3^s)^2}\right) \\
&= \frac{1}{2}\sqrt{\frac{1-3^s}{1+s^2}}\left(\frac{(1-3^s)2s+(1+s^2)(\ln 3)3^s}{(1-3^s)^2}\right)
\end{aligned}$$

12. Find $\frac{d^2}{dx^2}\left(2^{x^2}\right)$, ie compute the second derivative of 2^{x^2}. (It may help to think of the function as $2^{\left(x^2\right)}$).

$$\begin{aligned}
\frac{d}{dx}\left(2^{x^2}\right) &= (\ln 2)2^{x^2}\frac{d}{dx}(x^2) = (\ln 2)2^{x^2}(2x) = 2\ln(2)x2^{x^2} \\
\frac{d^2}{dx^2}\left(2^{x^2}\right) &= \frac{d}{dx}\left(2\ln(2)x2^{x^2}\right) = 2(\ln 2)\frac{d}{dx}\left(x2^{x^2}\right) \\
&= 2(\ln 2)\left(2^{x^2}+x(\ln 2)2^{x^2}(2x)\right) = 2(\ln 2)2^{x^2}+4x^2(\ln 2)^2 2^{x^2} \\
&= (\ln 2)2^{x^2+1}(1+2x^2(\ln 2))
\end{aligned}$$