

MATH 110 - SECTION 8

Quiz #4

Solutions

1. Find all asymptotes of $f(x) = \frac{x^2 - 2x - 3}{2x^2 - 2x - 12}$.

First we check for common factors by factoring the top and bottom,

$f(x) = \frac{(x+1)(x-3)}{2(x+2)(x-3)}$. $x-3$ is a common factor to the top and bottom, so the rational function isn't in reduced form. So we cancel to get $f(x) = \frac{x+1}{2(x+2)}$ when $x \neq 3$. $f(x)$ has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = \frac{1}{2}$.

2. Find the equation for a rational function with a vertical asymptote at $x = 1, 3$ and a horizontal asymptote at $y = -2$.

Starting with the vertical asymptotes, we know that the rational function must be of the form $\frac{g(x)}{(x-1)(x-3)}$ for some $g(x)$. Furthermore, it has a nonzero horizontal asymptote, so the degree of $g(x)$ must be 2 (the same as the degree of the bottom). The only condition left is to make sure that it has the appropriate horizontal asymptote, so the leading coefficient of $g(x)$ must be -2 . For example, one solution would be $\frac{-2x^2}{(x-1)(x-3)}$