Math 110 - Section 8
Exam #4 - Sample study problems

Very important: These problems do not outline everything that will or will not be on the exam! They should be similar, however the goal of the exam is to test understanding - not regurgitation.

1. True or false: $\sum_{n=1}^{200} \ln(n) = \ln(200!)$

   If we write out what the sum means,

$$\sum_{n=1}^{200} \ln(n) = \ln(1) + \ln(2) + \ln(3) + \ln(4) + \cdots + \ln(199) + \ln(200)$$

   We know that $\ln(a) + \ln(b) = \ln(ab)$, so we can combine terms

$$\ln(1) + \ln(2) + \ln(3) + \ln(4) + \cdots + \ln(199) + \ln(200) = \ln\left(1(2)(3)(4)\ldots(199)(200)\right)$$

   The product on the inside is just the definition of the factorial, so

$$\ln\left(1(2)(3)(4)\ldots(199)(200)\right) = \ln(200!)$$

   So the statement is true.

2. Compute $\sum_{n=1}^{90} \left(\frac{1}{10}n + 36(5^{-(n-1)})\right)$

   We start out by breaking up the sum,

$$\sum_{n=1}^{90} \frac{1}{10}n + \sum_{n=1}^{90} 36(5^{-(n-1)})$$

   The constant can come out, and the negative exponent isn't very useful as is,

$$\frac{1}{10}\sum_{n=1}^{90} n + 36\sum_{n=1}^{90} \left(\frac{1}{5}\right)^{n-1}$$

   We have formulas for both sums,

$$\frac{1}{10}\frac{(90)(91)}{2} + 36\frac{1-(\frac{1}{5})^{90}}{1-\frac{1}{5}} = 438.3$$

3. Consider the sequence defined by

$$b_n = \begin{cases} 3 & \text{for } n = 1 \\ (b_{n-1} - 1)^2 & \text{for } n = 2, 3, 4, \ldots \end{cases}$$

Find the 4th term of the sequence We're told from the problem that $b_1 = 3$, so we have to dig through a few computations.

$$\begin{aligned} b_2 &= (b_1 - 1)^2 = (3-1)^2 = 4 \\ b_3 &= (b_2 - 1)^2 = (4-1)^2 = 9 \\ b_4 &= (b_3 - 1)^2 = (9-1)^2 = 64 \end{aligned}$$

So the 4th term, b_4, is 64

4. What type of sequence is $11, 8, 5, 2, -1, \ldots$?

   It looks like the sequence is decreasing at a constant rate, so we can compute the difference between the terms: $11 - 8 = 3$, $8 - 5 = 3$, $5 - 2 = 3$, and $2 - (-1) = 3$, so the common difference is -3 and the sequence is an arithmetic sequence.

5. Find $40 + 20 + 10 + 5 + \cdots + \frac{5}{8}$ using sum formulas.

   First we need to decide what type of sum this is; all of the terms have the same ratio so it's a geometric sum ($\frac{20}{40} = \frac{10}{20} = \frac{5}{10} = \frac{1}{2}$). Before we can use any formulas we need a way to write out the nth term. We know that $a_n = 40(\frac{1}{2})^{n-1}$, but which term is $\frac{5}{8}$?

$$\frac{5}{8} = 40\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$$

   $n - 1 = 6$, so $n = 7$. In summation notation, $\sum_{k=1}^{7} 40(\frac{1}{2})^{n-1}$. This is a finite geometric sum, so by the formula,

$$\sum_{k=1}^{7} 40\left(\frac{1}{2}\right)^{n-1} = 40\sum_{k=1}^{7}\left(\frac{1}{2}\right)^{n-1} = 40\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}} = 80\left(1 - \left(\frac{1}{2}\right)^7\right) = 79.375$$

6. If $\sum_{k=1}^{42} 2c_k = 90$, what is the value of $\sum_{k=1}^{42}(3c_k - 1)$?

   We can pull the 2 out of the sum to get just the value of c_k:

$$\begin{aligned} \sum_{k=1}^{42} 2c_k &= 90 \\ 2\sum_{k=1}^{42} c_k &= 90 \\ \sum_{k=1}^{42} c_k &= 45 \end{aligned}$$

Now we just break up the sum we need to calculate:

$$\sum_{k=1}^{42}(3c_k - 1) = \sum_{k=1}^{42} 3c_k - \sum_{k=1}^{42} 1 = 3\sum_{k=1}^{42} c_k - 42 = 3(45) - 42 = 93$$

7. If b is a geometric sequence in which $b_2 = 15$ and $b_4 = \frac{27}{5}$, find the common ratio and find a formula for b_n:

   If r is the common ratio, we know that $b_3 = rb_2$, and $b_4 = rb_3$, so $b_4 = r^2 b_2$. $\frac{27}{5} = 15r^2$, so solving for r we get $r = \frac{3}{5}$ (or 0.6 if you prefere). $b_2 = b_1 r$, so $15 = \frac{3}{5}b_1$. We get $b_1 = 25$.

$$b_n = 25\left(\frac{3}{5}\right)^{n-1}$$

8. Compute $\sum_{k=1}^{\infty} 4\left(\frac{7}{8}\right)^{k-1}$

   First we note that the common ratio, r, is $\frac{7}{8}$. Then this is a direct application of the sum formula for infinite geometric sums:

$$\sum_{k=1}^{\infty} 4\left(\frac{7}{8}\right)^{k-1} = \frac{4}{1-\frac{7}{8}} = \frac{4}{\frac{1}{8}} = 4(8) = 32$$

9. Compute $2 + \frac{5}{2} + 3 + \frac{7}{2} + 4 + \cdots + 9$

   Each of the terms has a common difference of $\frac{1}{2}$, so this is an arithmetic sum. We can write down a formula for the nth term: $a_n = 2 + (n-1)\frac{1}{2} = \frac{3}{2} + \frac{1}{2}n$. Now we need to decide which term 9 is in our sum, so we substitute it's value into our formula and look for n:

$$\begin{aligned} 9 &= \frac{3}{2} + \frac{1}{2}n \\ \frac{15}{2} &= \frac{1}{2}n \\ n &= 15 \end{aligned}$$

   So we can use the formula for a sum of an arithmetic sequence,

$$\frac{15}{2}(2+9) = 82.5$$

10. Simplify the following expression using properties of the factorial:

$$\frac{k!(k^2+3k+2)}{9}$$

   If we factor the polynomial it becomes clear that this is a rather silly problem:

$$\frac{k!(k^2+3k+2)}{9} = \frac{k!(k+1)(k+2)}{9} = \frac{(k+2)!}{9}$$

11. Write $0.121212\overline{12}$ as a fraction

We can write the decimal as a sum, $\sum_{n=1}^{\infty}(0.12)(0.01)^{n-1}$. Then by the formula for infinite geometric sums,

$$\sum_{n=1}^{\infty}(0.12)(0.01)^{n-1} = \frac{0.12}{1-0.01} = \frac{0.12}{.99} = \frac{12}{99} = \frac{4}{33}$$