

Linear algebra HW 7

Jordan canonical form

- 1 Compute the Jordan canonical form for $A = \begin{pmatrix} 0 & 1 & -1 \\ -11 & 5 & -7 \\ -2 & -1 & -1 \end{pmatrix}$.

You should be able to find the roots of the minimal polynomial by looking at it.

- 2 Use Matlab or another computer algebra system to find the Jordan canonical form of

$$A = \begin{pmatrix} 1 & -12 & 0 & 8 & 3.5 \\ 0 & -6 & 1 & 4 & 2.5 \\ -2 & -3 & 2 & 4 & 2 \\ -0.5 & -8.5 & 1.5 & 7 & 2.75 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

- 3 Determine which of the following matrices are in Jordan canonical form:

$$\begin{pmatrix} 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}, \begin{pmatrix} 3 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

4] Let $A = \begin{pmatrix} 10 & -2 \\ 3 & 3 \end{pmatrix}$, diagonalize A and use your results to find a matrix B such that $B^2 = A$.

5] What information about a matrix can you read off of its Jordan canonical form? Can you think of something which you can't see from the Jordan canonical form?

6] If A is an 2×2 matrix with eigenvalues 2, 5 and B is a 2×2 matrix with eigenvalues 1, 10, is it possible that $A = C^{-1}BC$ for some matrix C ? Justify your answer.