Math 215 Project 2: Balancing Chemical Equations

Written by: Rebecca Strautman

Problem Description: Chemical equations are written with the reactants (the chemicals you start with) on the left, and the products (the chemicals you end with) on the right. By the Law of Conservation of Mass, the number of atoms must be the same on both sides (for instance, if you have 2 atoms of carbon on the left, you must have 2 atoms of carbon on the right) because these atoms cannot be created or destroyed in a reaction. Balancing an equation means figuring out how much of each reactant you need to make how much of each product. For example, to balance the equation:

$$H_2 + O_2 \to H_2O$$

We simply do the following:

$$2H_2 + O_2 \to 2H_2O$$

Now we have 4 atoms of H on each side, and 2 of O, so it is balanced. For many problems, you can probably figure the constants out in your head, but if there are many elements involved, we can instead use linear algebra.

Example Problem: Consider the unbalanced reaction

$$C_2H_6 + O_2 \to CO_2 + H_2O.$$

Assign each molecule a variable $x_1, \ldots, x_4$ since we have 4 expressions in the reaction.

$$x_1C_2H_6 + x_2O_2 \to x_3CO_2 + x_4H_2O.$$

Now, we write out the requirements for each element

$$\begin{aligned} C &: 2x_1 = x_3 \\ H &: 6x_1 = 2x_4 \\ O &: 2x_2 = 2x_3 + x_4. \end{aligned}$$

Bringing everything to the left hand side, we have a system of linear equations which we can write in the form

$$\begin{aligned} 2x_1 - x_3 &= 0 \\ 6x_1 - 2x_4 &= 0 \\ 2x_2 - 2x_3 - x_4 &= 0. \end{aligned}$$

We put these into an augmented matrix and get

$$\left[\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right].$$

The reduced row echelon form of this matrix is

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1/3 & 0 \\ 0 & 1 & 0 & -7/6 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \end{array}\right].$$

This gives the solution set

$$\mathbf{x} = \begin{bmatrix} 1 \\ \frac{3}{2} \\ \frac{7}{6} \\ \frac{2}{3} \\ 1 \end{bmatrix} t.$$

We need to choose t such that the vector $\mathbf{x}$ contains only integers, and so the components of $\mathbf{x}$ do not have any common factors (we want the equation to be in its reduced form). In this solution set, we can take $t = 6$. So our balanced chemical equation is

$$2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O.$$

Assigned Problem: Use the method described above to balance each of the following chemical equations:

(a) $K_4Fe\left(CN\right)_6 + H_2SO_4 + H_2O = K_2SO_4 + FeSO_4 + \left(NH_4\right)_2 SO_4 + CO$

(b) $PhCH_3 + KMnO_4 + H_2SO_4 = PhCOOH + K_2SO_4 + MnSO_4 + H_2O.$

A few helpful hints: Subscripts on an atom denote the number of atoms of that element in one molecule of the reactant or product. If multiple atoms are in parentheses with a subscript $\left(NO_3\right)_2$, this subscript distributes and you have 2 atoms of nitrogen (N) and 6 of oxygen (O). Finally, for those who are unfamiliar with chemistry, abbreviations for elements consist of either a capital letter (O, N, C...) or a capital letter followed by a lowercase one (Na, Cl, Ag...), so KCl consists of two elements, denoted by K and Cl.