Math 120 - Section 6
Exam #2

Instructors Name: Ryan Smith
Students Name (please print): `Solutions`

1. Expand and completely simplify the expression: $\ln\left(\frac{16y^4}{\sqrt{x^3z^5}}\right)$

$$\ln\left(\frac{16y^4}{\sqrt{x^3z^5}}\right) = \ln 16 + \ln y^4 - \frac{1}{2}(\ln x^3 + \ln z^5) = 4\ln 2 + 4\ln y - \frac{3}{2}\ln x - \frac{5}{2}\ln z$$

2. Richard Cory's accountants are trying to decide between two investment plans: the first would invest $\$32,000$ at 4% compounded monthly, the second would invest $\$28,000$ at 5% compounded continously. Setup an equation for each of the two investment options and explain how you would determine which is better after 2 years.

The equation for the first investment plan would be

$$A = 32000\left(1 + \frac{0.04}{12}\right)^{(12)(2)}$$

since it is discrete growth. For the second we use the formula for continuous growth,

$$A = 28000e^{0.05(2)}$$

In order to determine which option is the best investment choice we would use a calculator to compute the ratio of money after 2 years to the initial investment (after all, ending with 17,000 on an investment of 10,000 is a much better investment option than having 23,000 after you've invested 20,000).

3. Let $g(x) = \ln(6x + 12) - 1$, find $g^{-1}(x)$:
We switch x and y and solve

$$\begin{aligned} x &= \ln(6y + 12) - 1 \\ x + 1 &= \ln(6y + 12) \\ e^{x+1} &= 6y + 12 \\ \frac{1}{6}e^{x+1} - 2 &= y \\ g^{-1}(x) &= \frac{1}{6}e^{x+1} - 2 \end{aligned}$$

4. Solve for x in the following equation: $2\left(x^{\frac{1}{2}}+2\right)^3 = 100$

$$\begin{aligned}\left(x^{\frac{1}{2}}+2\right)^3 &= 50\\ x^{\frac{1}{2}}+2 &= \sqrt[3]{50}\\ x &= (\sqrt[3]{50}-2)^2\end{aligned}$$

5. The number of bacteria on a petri dish is doubling every 22 minutes. How long would an underpaid lab assistant have until there were 5 times the original number of bacteria?

`This is a continuous growth problem, so` $A = Pe^{kt}$ `for an appropriate constant` k. `Assuming that` t `measures the time in minutes,` $2P = Pe^{22k}$, `so` $k = \frac{\ln 2}{22}$. `We still don't know how many bacteria there were at the start of the experiment - and we don't need to! Setting the output to be 5 times the original,`

$$\begin{aligned}5P &= Pe^{\frac{\ln 2}{22}t}\\ \ln 5 &= \frac{\ln 2}{22}t\\ t &= \frac{22\ln 5}{\ln 2}\end{aligned}$$

6. Solve the equation $e^{3x} = 7^{2-x}$ using logs.

$$\begin{aligned}\ln e^{3x} &= \ln 7^{2-x}\\ 3x &= (2-x)\ln 7\\ 3x + x\ln 7 &= 2\ln 7\\ x &= \frac{2\ln 7}{3+\ln 7}\end{aligned}$$

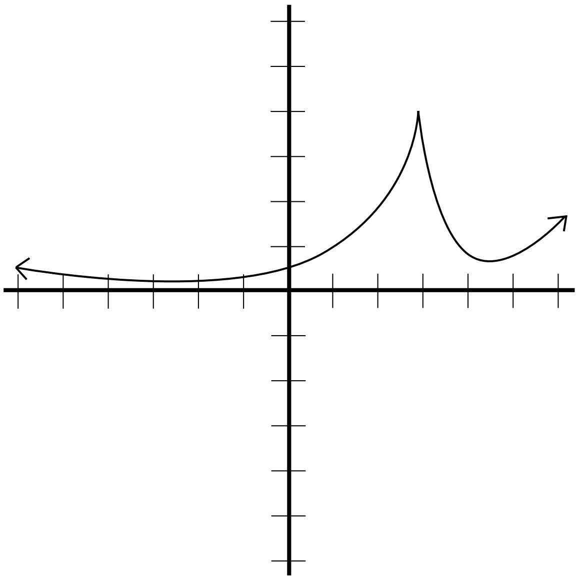
7. Which of the following statements is / are true? For those that are true give a brief explaination of why they are true; for those that are false provide an example to show why they are false.
   I. $\ln(x) = -\ln\left(\frac{1}{x}\right)$ for all x: True, provided $x > 0$ since $\frac{1}{x} = x^{-1}$
   II. $\dfrac{\log(y)}{\log(z)} = \log(y - z)$: False, division inside is subtraction outside.
   III. $\ln(x)$ is only defined for $x > 1$: False, $\ln(\frac{1}{2}) = -\ln 2$

8. When a chemical plant blows a gasket and covers the land in industrial solvents, the government begins a rapid cleanup program. During the first year they remove 35% of the pollution, but as people loose interest the cleanup slows down. Each year they cleanup 35% of the pollution left over from the previous year. If 400 metric tons of pollutant were spilled, write a function to model the amount of pollution left after t years. Use this formula to solve for the amount of pollution after 8 years (you may leave your answer in terms of logs).

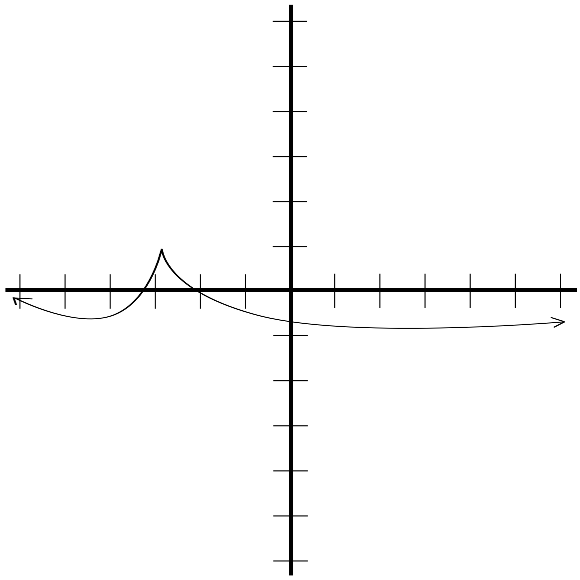
   This is just a basic application of the discrete growth formula, where the rate of growth is -35% per year. Here $n = 1$ since the clean up is once per year.

$$
\begin{aligned}
A &= P(1 + r)^t \\
A &= 400(1 - 0.35)^8 \\
A &= 400(0.65)^8
\end{aligned}
$$

9. The graph of the function $T(z)$ is given below. Sketch the graph of $\frac{1}{2}T(-z) - 1$. You may wish to note that $T(1) = 1$ and $T(3) = 4$.



$y = T(z)$



$y = \frac{1}{2}T(-z) - 1$

10. What is the domain of $h(x) = \ln(2 - x) + e$:
In order for the log to be defined the inside of the parentheses must be greater than zero, $2 - x > 0$. That is the same as requiring that $x < 2$, so the domain is $(-\infty, 2)$.

11. The problem concerns even and odd functions (it may help to draw a graph).

    (a) If we know that $f(x)$ is an even function, do we know what $f(0)$ is? If so find $f(0)$, otherwise explain why we don't have enough information to say:
    `Impossible to say, consider the constant function` $f(x) = c$. c `could be any number and the function is still even.`

    (b) If we know that $f(x)$ is an odd function, do we know what $f(0)$ is? If so find $f(0)$, otherwise explain why we don't have enough information to say.
    `According to the definition of an odd function,` $f(-x) = -f(x)$ `for all` x. `This means that` $f(-0) = -f(0)$, `so` $f(0) = -f(0)$. `Clearly the only value that works if` $f(0) = 0$ `(you can also just solve for` $f(0)$ `in the equation).`

12. Let $g(x)$ be an exponential function which passes through the points $(1, \frac{4}{3})$ and $(3, 3)$. Find the equation for $g(x)$.
    `The function will be of the form` $g(x) = a(b^x)$, `first we need to find` b.

$$
\begin{aligned}
3 &= \frac{4}{3}b^{3-1} \\
\frac{9}{4} &= b^2 \\
b &= \frac{3}{2}
\end{aligned}
$$

`Now we find` a `by plugging in a known point,`

$$
\begin{aligned}
3 &= a\left(\frac{3}{2}\right)^3 \\
a &= \frac{8}{9}
\end{aligned}
$$

`The final answer`

$$
g(x) = \frac{8}{9}\left(\frac{3}{2}\right)^x
$$

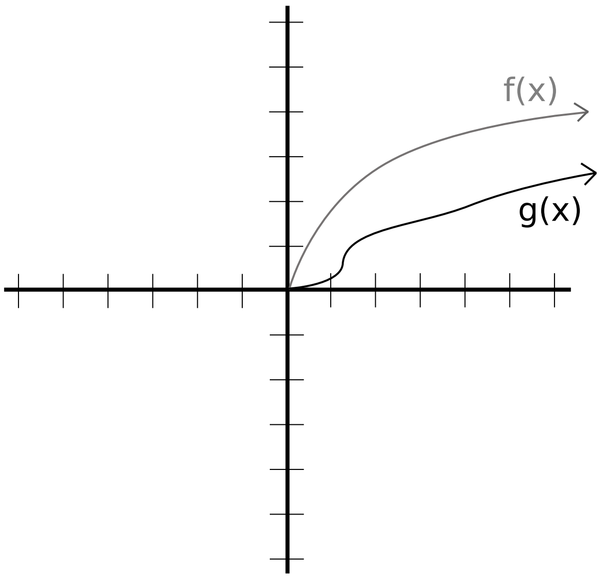
13. Let $f(x)$ and $g(x)$ be the functions defined by the tables of value below. $g(x)$ is a transformation of $f(x)$, determine what that transformation is (example, shifted right 2 and down 4). Use this to write $g(x)$ as a function of $f(x)$.

x	-3	-2	-1	0	1	2	3
f(x)	0	5	1	2	-2	-1	2

x	-3	-2	-1	0	1	2	3
g(x)	-4	2	4	-4	-2	-10	0

$g(x) = -2f(-x)$

14. Let $f(x)$ and $g(x)$ be the functions shown below, then $f(x) \geq g(x)$ for all x.



Will $f^{-1}(x) \geq g(x)$? Will $g^{-1}(x) \geq f^{-1}(x)$? Is it impossible to say? **Explain your answer.**

`To find the inverse graphically, you flip the graph of the function across the line` $y = x$ `(this switches the` x `and` y `coordinates). So if` $f(x) \geq g(x)$ `when they are flipped across that line` $g^{-1}(x) \geq f^{-1}(x)$.