Name: ____

Log Review Problems - Solutions

Combine into one logarithm:
1) $\log 60 - \log 4 = \log\left(\frac{60}{4}\right) = \log 15$
2) $\log(a+b) - \log(a-b) = \log\left(\frac{a+b}{a-b}\right)$
3) $\log p + 2\log r - \log 2 = \log\left(\frac{pr^2}{2}\right)$
4) $\log 1 + \log 2 + \cdots + \log(n-1) + \log n = \log(1(2)(3)\ldots(n-2)(n-1)n) = \log(n!)$
Expand the logarithm completely:
1) $\log ab^3 = \log a + 3\log b$
2) $\log 9b^{\frac{4}{5}}(1+a)^3 = 3\log 2 + \frac{4}{5}\log b + 3\log(1+a)$
3) $\log \dfrac{a^3}{\sqrt[3]{b^2c}} = 3\log a - \frac{2}{3}\log b - \frac{1}{3}\log c$
4) $\log \sqrt[4]{\dfrac{a^3\sqrt{c}}{b}} = \frac{1}{4}(3\log a + \frac{1}{2}\log c - \log b) = \frac{3}{4}\log a + \frac{1}{8}\log c - \frac{1}{4}\log b$

Properties of logs: If you know that $\log_b 2 = m$, $\log_b 3 = n$, $\log_b 5 = r$, and $\log_b 7 = s$ then write the following logs in terms of m, n, r and s.
9) $\log_b \frac{2}{3} = m - n$
10) $\dfrac{\log_b 2}{\log_b 3} = \dfrac{m}{n}$
11) $\log_b 2^2 = 2m$
12) $(\log_b 2)^2 = m^2$
13) $\log_b 30 = \log 2 + \log 3 + \log 5 = m + n + r$
14) $\log_b 350 = \log 2 + 2\log 5 + \log 7 = m + 2r + s$
15) $\log_b \frac{70}{b} = \log 2 + \log 5 + \log 7 - 1 = m + r + s - 1$
16) $\log_b \dfrac{1}{3} = -\log 3 = -n$
17) $(\log_b 2)(\log_b 3) = nm$

True or false

- It is possible to take the log of a negative number. `False`
- A log can be zero. `True`
- $\log(a+b) = \log(a)\log(b)$ `False`

Solve the equation for x:

1) $\log(x+1) = \log(2x)$

$x+1 = 2x$, so $x = 1$

2) $\log_5 x = 2\log_5 3$

$\log_5 x = \log_5 9$, so $x = 9$

3) $\log_2 x^2 = 3 + \log_2 x$

$2\log_2 x = 3 + \log_2 x$, so $\log_2 x = 3$, therefore $x = 8$

4) $5(3^x) = 4^x$

$\log 5 + x\log 3 = x\log 4$, so $x = \frac{\log 5}{\log 4 - \log 3}$

5) $3\log_2 x + \log_2 27 = 3$

$\log_2 x = 1 - \log_2 3$, so $\log_2(3x) = 1$, therefore $x = \frac{2}{3}$

Advanced Problems:

Simplify the expression:

1) $\log_{49} 7 - \log_8 64 = \frac{1}{2} - 2 = -\frac{3}{2}$

2) $\dfrac{\log_3 \sqrt{243\sqrt{81\sqrt[3]{3}}}}{\log_2 \sqrt[4]{64} + \ln e^{-10}} = \dfrac{\log_3 \sqrt{3^4\sqrt{3^3 3^{\frac{1}{3}}}}}{\frac{1}{4}\log_2 2^6 - 10} = \dfrac{\log_3 \sqrt{3^4 3^{\frac{2}{3}}}}{\frac{3}{2} - 10} = \dfrac{\log_3 3^{\frac{14}{3}}}{-\frac{17}{2}} = \dfrac{\frac{14}{3}}{-\frac{17}{2}} = -\frac{28}{51}$

Solve for x and completely simplify your answer:

3) $\log_{x-1}(4x-4) = 2$

$$\begin{aligned}
\log_{x-1}(4x-4) &= 2 \\
\frac{\ln(4x-4)}{\ln(x-1)} &= 2 \\
\ln(4x-4) &= 2\ln(x-1) \\
4x-4 &= (x-1)^2 \\
(x-1)(x-5) &= 0 \\
& x = 1, 5 \text{ but } x = 1 \text{ is not solution if you check the equation, so } x = 5
\end{aligned}$$

4) $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left[\left(\frac{1}{a} - a\right)^2\right]$

$$\begin{aligned}
\log_b x &= \log_b(1-a) + \log_b(1+a) - \log_b\left(\frac{1-a}{a}\right) \\
\log_b x &= \log_b(1-a) + \log_b(1+a) - \log_b(1-a) + \log_b a \\
\log_b x &= \log_b(1+a) + \log_b a \\
x &= a + a^2
\end{aligned}$$

5) $\log_b x = 2 - a + \log_b\left(\frac{a^2 b^a}{b^2}\right)$

$$
\begin{aligned}
\log_b x &= 2 - a + \log_b\left(\frac{a^2 b^a}{b^2}\right) \\
\log_b x &= 2 - a + 2\log_b a + a - 2 \\
\log_b x &= 2\log_b a \\
x &= a^2
\end{aligned}
$$