Math 120R
Trigonometry Lab
Hours of Daylight

As you have probably noticed, over the course of a year, the length of the day or the number of hours of daylight hours does not remain the same. The following table gives the number of daylight hours (rounded to the nearest tenth of an hour) for Boston, Massachusetts. The latitude for Boston is 42 degrees North.

Date	Day	Daylight hours	Date	Day	Daylight hours
12/31	0	9.1	7/9	190	15.1
1/10	10	9.3	7/19	200	14.9
1/20	20	9.6	7/29	210	14.6
1/30	30	9.9	8/8	220	14.2
2/9	40	10.3	8/18	230	13.8
2/19	50	10.8	8/28	240	13.3
3/1	60	11.2	9/7	250	12.9
3/11	70	11.7	9/17	260	12.4
3/21	80	12.2	9/27	270	11.9
3/31	90	12.7	10/7	280	11.5
4/10	100	13.1	10/17	290	11.0
4/20	110	13.6	10/27	300	10.6
4/30	120	14.0	11/6	310	10.1
5/10	130	14.4	11/16	320	9.8
5/20	140	14.8	11/26	330	9.5
5/30	150	15.2	12/6	340	9.2
6/9	160	15.3	12/16	350	9.1
6/19	170	15.3	12/26	360	9.1
6/29	180	15.3			

Plot these points on your calculator with day of the year, t, as the independent variable and number of hours of daylight, H, as the dependent variable. The shape you get should look approximately like a sinusoidal function. As you should recall from the text, a curve that is sinusoidal can be modeled by either a sine or cosine function. For consistency, use the cosine function. You should find a function of the form

$$H = f(t) = A\cos(B(t + C)) + D.$$

You need to begin by finding values for *A, B, C* and *D.* Recall from the text that these letters are called *parameters* and they represent different quantities depending on the data given. Refer to chapter 6.5 in the text for a review of what these parameters represent in a mathematical and practical sense. In your lab report you should show all the work involved in finding these parameters and explain what each one means in both a mathematical and practical sense. Note that while the data is only recorded every 10 days you must remember that there are generally 365 days in the year when you write your equation. Also be sure your calculator is in radian mode. After you have come up with an equation you should graph your equation along with your data on your graphing calculator. Remember the data will not exactly fit the equation but they should be very close to one another. If they are not close you will need to make some adjustments to your equation.

Now do the same thing for Fairbanks, Alaska which has latitude 64N. Fairbanks is just south of the Arctic Circle. The data is as follows:

Date	Day	Daylight hours	Date	Day	Daylight hours
12/31	0	3.7	7/9	190	20.6
1/10	10	3.9	7/19	200	20.3
1/20	20	4.2	7/29	210	18.8
1/30	30	5.7	8/8	220	18.4
2/9	40	8.2	8/18	230	15.9
2/19	50	8.6	8/28	240	15.5
3/1	60	9.1	9/7	250	15.0
3/11	70	10.7	9/17	260	13.4
3/21	80	12.2	9/27	270	11.0
3/31	90	13.7	10/7	280	10.5
4/10	100	15.3	10/17	290	8.9
4/20	110	15.7	10/27	300	8.4
4/30	120	16.2	11/6	310	8.0
5/10	130	18.6	11/16	320	5.6
5/20	140	20.2	11/26	330	4.0
5/30	150	20.5	12/6	340	3.8
6/9	160	20.6	12/16	350	3.7
6/19	170	20.7	12/26	360	3.7
6/29	180	20.7			

Determine an equation for this data like you did with the data for Boston. Remember, you will also need to explain what the quantities represent in a practical sense as you did with the equation you found for Boston.

Lab Report

Your lab report should include the equations for Boston and Fairbanks, how you derived these equations, the graphs of the equations and what all the parameters in the equations represent in a practical sense.

Now let's delve into what this data is telling you. Why are your equations different? What are the differences and what accounts for these differences? How would the parameters change in your equations if you were given this data for Tromso, Norway which has latitude of 69 N and is a bit more than 200 miles inside the Arctic Circle? You should also discuss the relevance of the Arctic Circle. How would the parameters change if the city was Singapore which has latitude 1 N? How would the parameters change if the city were Wellington, New Zealand which is approximately as far south of the equator as Boston is north of the equator (i.e. their latitude is 42 S)? Explain you reasoning and write an approximate equation for all of these cities.

Using the length of day function you wrote for Boston and the graph, find the season of the year when the days are lengthening the most rapidly. What is this point on the graph called in a mathematical sense? Zoom in on this section until the graph looks like a line (this is called local linearity). Now use the trace feature on your calculator to find two coordinate points on this function and calculate the slope of the line through these two points. Use this slope to determine how rapidly the days are lengthening at that time of year. Now convert your calculations into minutes per day and minutes per week. Repeat this for the Fairbanks function. Compare the results of Fairbanks with Boston. Explain the difference between these two numbers.