Fall 2007 Math 120, Section 5

Final Exam Practice

Disclaimer: These problems are not intended to be a comprehensive review of the course. You should review all your class notes, homework (collected and uncollected), previous exams, previously suggested review problems, etc.

1 Problem One

For each of the following functions, state whether the function is linear, quadratic, a polynomial (and state the degree if so), power, rational, exponential, logarithmic, trigonometric, a composition of any of these, or none of these.

1. $f(x) = \dfrac{x^2(x^3-1)^{1/2} - x^{-1}(x^3-1)^{-1/2}}{\sqrt{x^3-1}}$.
2. $f(x) = 3\cos^2 x + 3\sin^2 x + x^2 - 2x + 1$.
3. $f(x) = 3 \cdot 2^x$
4. $f(x) = 2\sin 2x$
5. $f(x) = 3\log_b(x-3) + 7$
6. $f(x) = 3$
7. $f(x) = 3x^4 - 5x^2 + \dfrac{1}{x^{-3}} - \dfrac{1}{7}$.
8. $f(x) = \pi x^{\mathrm{e}}$
9. $f(x) = \sqrt{x-3}$
10. $f(x) = x^{\pi}$
11. $f(x) = x^x$
12. $f(x) = \mathrm{e}^{\mathrm{e}^{\mathrm{e}^x}}$

2 Problem Two

The expression $n!$ for a positive integer n means $n(n-1)(n-2)\cdots(2)(1)$ and is read "n-factorial," and we simply define $0! = 1$. Simplify the following expressions:

1. $\dfrac{2^{n+1}(n+1)!}{2^n n!}$.
2. $\displaystyle\sum_{i=1}^{n} \ln i$.

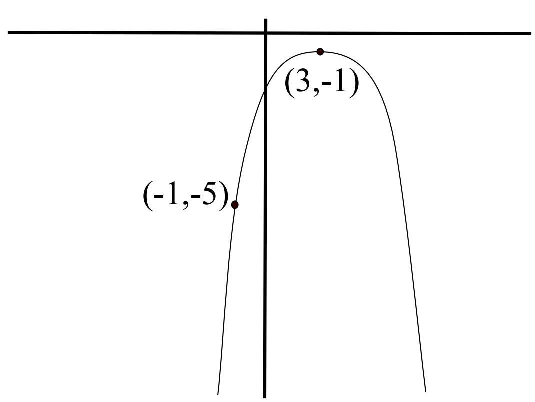
3 Problem Three

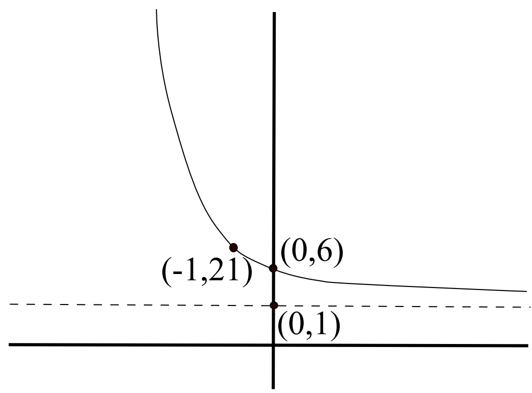
Graph each of the following functions. Label all relevant features (asymptotes, intercepts, etc.)

1. $f(x) = P_0 e^{\alpha x} - \beta$ where P_0, α and β are fixed positive constants.
2. $f(x) = 2x^2 - 5x + 3$
3. $f(x) = 2\ln(x-1) + 3$
4. $f(x) = -2(x^2 - 25)(x+4)^2(x-7)^5$
5. $f(x) = -3x + 7$
6. $f(x) = 10,674$
7. $f(x) = 3\sin(2\pi(x-3)) + 7$
8. $f(x) = \sec^2 x - \tan^2 x - 1$
9. $f(x) = \dfrac{x^2 - 5x + 6}{2x^2 - 12x + 16}$
10. $f(x) = 7\cos 6(x+2) - 2$
11. $f(x) = x^3 - 2x^2 + x$
12. $f(x) = \dfrac{-2x^2 + 1}{x - 1}$
13. $f(x) = 3e^{-x}$
14. $f(x) = 2\log_2(-x)$

4 Problem Four

Find a possible formula for each of the following graphs:

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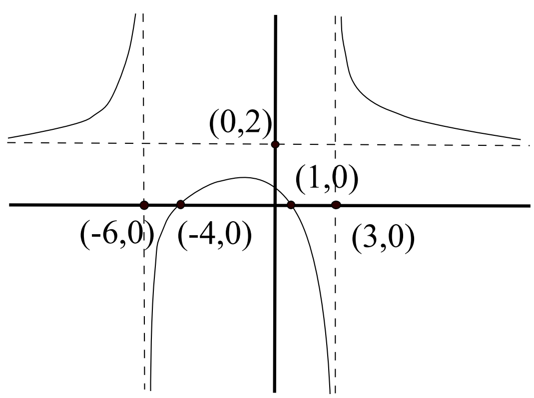
(-1,21)
(0,1)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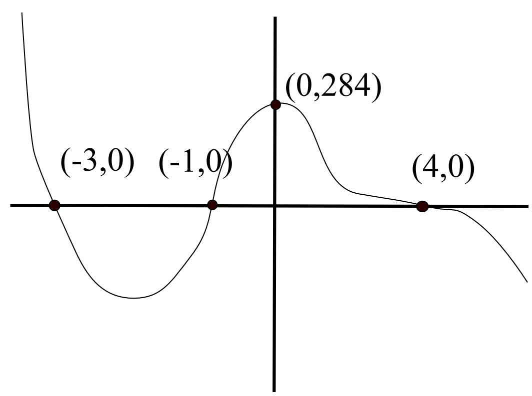
(1,0)
(-6,0)
(-4,0)
(3,0)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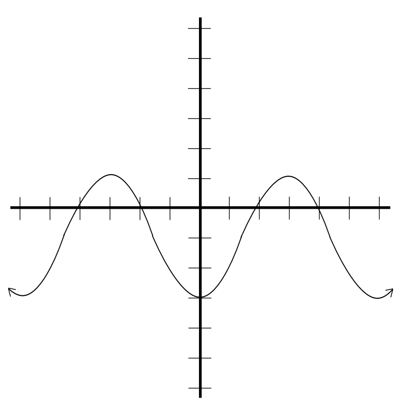
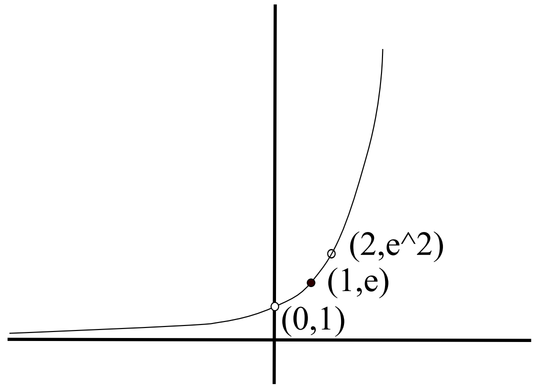
(-3,0)
(-1,0)
(4,0)



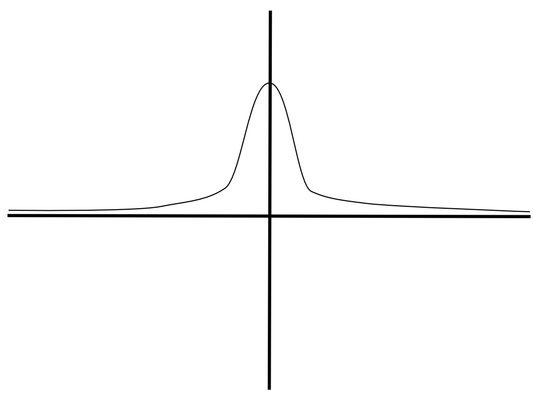


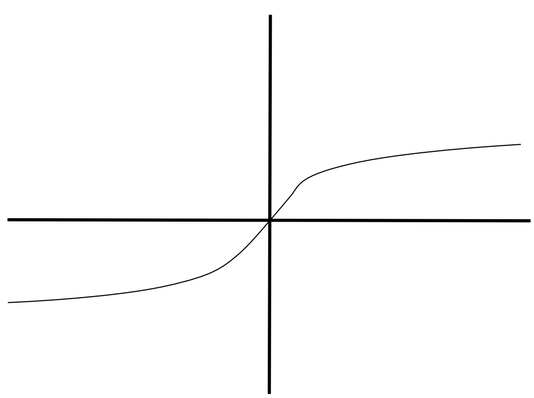
5 Problem Five

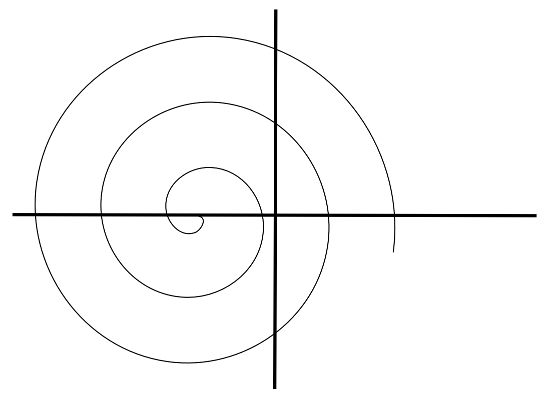
Which of the following represent functions? For those that represent functions, which might have inverse functions? Justify your answers.

t	-7	11	17	37	101
α	123	983	456	98	π

y	- e	$\pi + \mathrm{e}$	e^{π}	5670	$\mathrm{e}^{\mathrm{e}^{\mathrm{e}}}$	57289141
x	-1	345	2/7	- 1	π	$\pi + \mathrm{e}$







α	-2	-1	0	-1
β	23	81	1	0

6 Problem Six

Find the domain of each of the following functions.

1. $f(x) = \dfrac{x-1}{x^2-2x+5}$
2. $f(x) = \ln(x-5)$
3. $f(x) = \log_2(x-5)$
4. $f(x) = \log_b(x-5)$
5. $f(x) = \sqrt{x^2-9}$
6. $f(x) = 3\log_7(2x-7)+5$
7. $f(x) = \pi x^{\mathrm{e}} - 3x^{\pi}$
8. $f(x) = x^x$
9. $f(x) = \dfrac{(x-1)(x+2)}{(x-1)(x+3)}$
10. $f(x) = \dfrac{\sqrt{x-3}}{x+2}$
11. $f(x) = \dfrac{x}{x}$
12. $f(x) = \dfrac{x-5}{\sqrt{5-x}}$
13. f is a rational function with vertical asymptotes at $x = -2267, 2001,$ and 1953, holes at $x = -5413$ and $x = -\pi$, a zero at $x = \pi^{\mathrm{e}}$ and a horizontal asymptote given by $y = 1234$.

7 Problem Seven

Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for each of the following functions.

1. $f(x) = \dfrac{1}{x^2} + \sqrt{2}$.
2. $f(x) = \sqrt{x+7}$
3. $f(x) = 3$
4. $f(x) = \dfrac{1}{x-3}$
5. $f(x) = 3x^2 - 2x + 79134$
6. $f(x) = mx + b$ with $m \neq 0$.

8 Problem Eight

Write each of the following functions as compositions of nontrivial functions in as many ways as you can think of. Note that a trivial function is a function of the form $f(x) = x$ (or the same function with a different letter for f and a different letter for x).

1. $r(t) = \mathrm{e}^{\mathrm{e}^{\sin t}} - \cos\left(\mathrm{e}^{\sin t}\right)$
2. $s(z) = \dfrac{\ln\left(z^2+z-1\right)}{\mathrm{e}^{z^2+z-1}} - z^2 - z + 1.$
3. $f(x) = c$ where c is a constant
4. $q(p) = p^2 + 2p + 1$
5. $m(\zeta) = \dfrac{\zeta - 3}{\zeta + 4}$
6. $d(y) = \sin\left(\mathrm{e}^{\dfrac{y^2-1}{y^2+2}}\right)$

9 Problem Nine

Simplify the following expressions.

1. $\dfrac{3z^2(z^2+16)^{1/2} - z^4(z^2+16)^{-1/2}}{z^2+16}$
2. $\log_b\left(\dfrac{b^7\sqrt[3]{(xy)^3+8}}{(xy)^3}\right)$ (expand)

3. $\dfrac{(p^2+4)^{1/2}}{p^2+4}$

4. $\log_b(23) + 3\log_b(x-3) - 5\log_b(\sqrt{z+34}) + 7\log_b(y) - b$ (contract)

5. $\ln(P_0 e^{-\alpha x})$

10 Problem Ten

Find a formula for a function f satisfying the specified conditions.

1. f is a rational function with vertical asymptotes at $x = -4$ and $x = -1$, a hole at $x = 1$, no x-intercepts and a horizontal asymptote given by $y = 2$.

2. f is a linear function that has an x-intercept at $x = -3$ and y-intercept at $y = 3$.

3. f is a degree 7 with a zero of multiplicity 5 at $x = 1$, a zero of multiplicity 1 at $x = 4$, a zero of multiplicity 1 at $x = 3$ and a y-intercept 36.

4. f is a linear function whose graph passes through the points $(-2, -18)$ and $(1, -2/3)$.

5. f is an exponential function such that $f(-2) = -18$ and $f(1) = -2/3$.

6. f is a linear function parallel to the line $2x - 3y + 12 = 0$ passing through $(1, 3)$. Repeat with ‘parallel’ replaced by ’perpendicular’.

7. f is an exponential function with horizontal asymptote given by $y = 3$ whose graph passes through the points $(0, 6)$ and $(2, 15)$.

8. f is a power function with $f(1) = 5$ and $f(5) = 625$.

11 Problem Eleven

Solve the following equations.

1. $2x^2 - 3x + 5 = -11x - 12$

2. $2(b-1) + 3(b-2) = 2b + 5(b+3)$

3. $e^{2y} - 5e^y = -6$.

4. $3 \cdot 2^{3z} - 7 = 5$

5. $P = \dfrac{1+rt}{1-rt}$ (solve for r)

6. $\log_b(x-3) - \log_b(x+3) = 0$

7. $e^{\ln x} = 2$

8. $e^{\ln x} = -2$ (think carefully!)

9. $\log_2(x) + \log_2(x+1) = 2$.

10. $(x-1)^2 = -1$ (think carefully!)

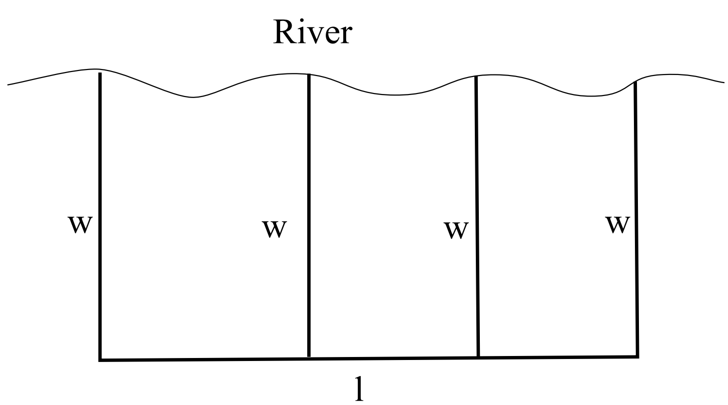
12 Problem Twelve

For each of the following functions $y = f(x)$, find a formula for the inverse function $f^{-1}(x)$. If necessary, restrict the domain of f (and state what that restriction is!)

1. $f(x) = 2x + 7$
2. $f(x) = 2x^2 - 3x + 5$ (hint - complete the square)
3. $f(x) = 2\log_b(2x + 1) - 7$
4. $f(x) = -3\sin(2\pi(x - 1)) + 5$
5. $f(x) = 3 + 2e^{-\lambda x}$ where $\lambda > 0$ is a fixed constant.
6. $f(x) = \dfrac{2x - 1}{4x + 7}$
7. $f(x) = 2(x - 7)^3$
8. $f(x) = \sqrt[5]{x + 1}$
9. $f(x) = 5\cos(\pi x) + 1$
10. $f(x) = 14 \cdot 2^{3x} - 12$

13 Problem Thirteen

A rancher in Bisbee would like to enclose a rectangular region along a river. He has 900 yards of fencing. He wants to subdivide the region into three subregions. The completed fence should be as in the diagram below:



where w represents width and l represents length. Find the formula for a function that determines the total area enclosed as a function of **length**. Explain why this function has a maximum. Determine the length and width that will maximize the total enclosed area. Determine the maximum total area that can be enclosed.

14 Problem Fourteen

Determine which of the following functions are even, which are odd, and which are neither even nor odd.

1. $f(z) = z^2 - 2z + 1$
2. $r(\theta) = 2\cos\theta$
3. $s(t) = \dfrac{1 - \sin^2 t}{1 + \sin^2 t}$
4. $w(\alpha) = \mathrm{e}^{\alpha}$
5. $q(\xi) = \sin\xi \cos\xi$

15 Problem Fifteen

1. Find a formula for the n^{th} term of the sequence $1, 5, 9, 13, 17, \ldots$.
2. Use summation notation to write down the sum of all the terms between $n = 129$ and $n = 1,987$ of the sequence in the previous problem.
3. Find a formula for the n^{tg} term of the sequence $5, -10, 20, -40, 80, \ldots$.
4. Use summation notation to write down the sum of all the terms between $n = 12$ and $n = k$ (for some $k > 12$) of the sequence in the previous problem.

16 Problem Sixteen

Consider the function ζ defined by the following table of values.

t	0	5	10	15	20
$\zeta(t)$	20	40	70	100	110

Compute the average rates of change on the following intervals:

1. $0 \leq t \leq 5,$
2. $5 \leq t \leq 10,$
3. $10 \leq t \leq 15,$ and
4. $15 \leq t \leq 20.$

Is ζ concave up or concave down on the interval $0 \leq t \leq 10$? How about the interval $10 \leq t \leq 20$?

17 Problem Seventeen

The angle addition formulas for sine and cosine are as follows:

$$\begin{aligned}\sin(\theta+\phi) &= \sin(\theta)\cos(\phi)+\sin(\phi)\cos(\theta)\\ \cos(\theta+\phi) &= \cos(\theta)\cos(\phi)-\sin(\theta)\sin(\phi)\end{aligned}$$

Use the angle addition formulas to verify the following:

1. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
2. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$
3. $\tan(\theta+\phi) = \dfrac{\tan(\theta)+\tan(\phi)}{1-\tan(\theta)\tan(\phi)}$
4. $\tan(2\theta) = \dfrac{2\tan(\theta)}{1-\tan^2(\theta)}$

Use the angle addition formulas and the identities verified above to simplify the following:

1. $\dfrac{\sin(2\beta)}{\sin(\beta)}$
2. $\dfrac{\cos(2\gamma)}{\cos\gamma - \sin\gamma}$

Use the angle addition formulas to verify each of the following:

1. $\cos u - \cos v = -2\sin\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$
2. $\sin u + \sin v = 2\sin\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$
3. $\sin u - \sin v = 2\cos\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$
4. $\dfrac{\sin(x+h)-\sin x}{h} = \sin x\left(\dfrac{\cos(h)-1}{h}\right) + \cos x\left(\dfrac{\sin h}{h}\right).$
5. $\dfrac{\cos(x+h)-\cos x}{h} = \cos x\left(\dfrac{\cos(h)-1}{h}\right) - \sin x\left(\dfrac{\sin(h)}{h}\right).$

18 Eighteen

Determine which of the following are true and which of the following are false. Justify your answers. If possible, correct false statements.

1. $\cos t + \sin t = 1$
2. $\sqrt[3]{x^3+8} = x+2$

3. $(2y+4)^{10} = 1024(y+2)^{10}$
4. Every polynomial has at least on x-intercept.
5. Every function can have at most one y-intercept.
6. $\cos^2(f(\zeta)) + \sin^2(f(\zeta)) = 1$.
7. Every rational function has vertical asymptotes, holes, or both.
8. Every polynomial has a y-intercept.
9. If f is a function, $f(a+b) = f(a) + f(b)$.
10. $1 + \tan^2\beta = \sec^2\alpha$.
11. Every function is either even or odd.
12. Every rational function has an x-intercept.
13. A graph with more than one y-intercept represents a function, but it would not be one-to-one and therefore would not have an inverse.
14. $1 + \cot^2\theta = \csc^2\theta$
15. Every rational function has a y-intercept.
16. A function can be concave up only if it is increasing.
17. The graph of every straight line represents a function.
18. The graph of every straight line that represents a function actually represents a one-to-one function.
19. $\tan = \dfrac{\sin}{\cos}$.
20. $\log_b(M) - \log_b(N) = \dfrac{\log_b(M)}{\log_b(N)} = \log_b\left(\dfrac{M}{N}\right)$.