

# MATH 110 - SECTION 3

## Exam #4 - Sample study problems

**Very important:** These problems do not outline everything that will or will not be on the exam! They should be similar, however the goal of the exam is to test understanding - not regurgitation.

1. True or false:  $\sum_{n=1}^{200} \ln(n) = \ln(200!)$

If we write out what the sum means,

$$\sum_{n=1}^{200} \ln(n) = \ln(1) + \ln(2) + \ln(3) + \ln(4) + \cdots + \ln(199) + \ln(200)$$

We know that  $\ln(a) + \ln(b) = \ln(ab)$ , so we can combine terms

$$\ln(1) + \ln(2) + \ln(3) + \ln(4) + \cdots + \ln(199) + \ln(200) = \ln(1(2)(3)(4) \dots (199)(200))$$

The product on the inside is just the definition of the factorial, so

$$\ln(1(2)(3)(4) \dots (199)(200)) = \ln(200!)$$

So the statement is true.

2. Compute  $\sum_{n=1}^{90} \left(\frac{1}{10}n + 36(5^{-(n-1)})\right)$

We start out by breaking up the sum,

$$\sum_{n=1}^{90} \frac{1}{10}n + \sum_{n=1}^{90} 36(5^{-(n-1)})$$

The constant can come out, and the negative exponent isn't very useful as is,

$$\frac{1}{10} \sum_{n=1}^{90} n + 36 \sum_{n=1}^{90} \left(\frac{1}{5}\right)^{n-1}$$

We have formulas for both sums,

$$\frac{1}{10} \frac{(90)(91)}{2} + 36 \frac{1 - \left(\frac{1}{5}\right)^{90}}{1 - \frac{1}{5}} = 438.3$$

3. Consider the sequence defined by

$$b_n = \begin{cases} 3 & \text{for } n = 1 \\ (b_{n-1} - 1)^2 & \text{for } n = 2, 3, 4, \dots \end{cases}$$

Find the 4th term of the sequence We're told from the problem that  $b_1 = 3$ , so we have to dig through a few computations.

$$\begin{aligned} b_2 &= (b_1 - 1)^2 = (3 - 1)^2 = 4 \\ b_3 &= (b_2 - 1)^2 = (4 - 1)^2 = 9 \\ b_4 &= (b_3 - 1)^2 = (9 - 1)^2 = 64 \end{aligned}$$

So the 4th term,  $b_4$ , is 64

4. What type of sequence is 11, 8, 5, 2, -1, ...?

It looks like the sequence is decreasing at a constant rate, so we can compute the difference between the terms:  $11 - 8 = 3$ ,  $8 - 5 = 3$ ,  $5 - 2 = 3$ , and  $2 - (-1) = 3$ , so the common difference is 3 and the sequence is an arithmetic sequence.

5. Find  $40 + 20 + 10 + 5 + \dots + \frac{5}{8}$  using sum formulas.

First we need to decide what type of sum this is; all of the terms have the same ratio so it's a geometric sum ( $\frac{20}{40} = \frac{10}{20} = \frac{5}{10} = \frac{1}{2}$ ). Before we can use any formulas we need a way to write out the  $n$ th term. We know that  $a_n = 40(\frac{1}{2})^{n-1}$ , but which term is  $\frac{5}{8}$ ?

$$\frac{5}{8} = 40 \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$$

$n - 1 = 6$ , so  $n = 7$ . In summation notation,  $\sum_{k=1}^7 40(\frac{1}{2})^{n-1}$ . This is a finite geometric sum, so by the formula,

$$\sum_{k=1}^7 40 \left(\frac{1}{2}\right)^{n-1} = 40 \sum_{k=1}^7 \left(\frac{1}{2}\right)^{n-1} = 40 \frac{1 - (\frac{1}{2})^7}{1 - \frac{1}{2}} = 80 \left(1 - \left(\frac{1}{2}\right)^7\right) = 79.375$$

6. If  $\sum_{k=1}^{42} 2c_k = 90$ , what is the value of  $\sum_{k=1}^{42} (3c_k - 1)$ ?

We can pull the 2 out of the sum to get just the value of  $c_k$ :

$$\begin{aligned} \sum_{k=1}^{42} 2c_k &= 90 \\ 2 \sum_{k=1}^{42} c_k &= 90 \\ \sum_{k=1}^{42} c_k &= 45 \end{aligned}$$

Now we just break up the sum we need to calculate:

$$\sum_{k=1}^{42} (3c_k - 1) = \sum_{k=1}^{42} 3c_k - \sum_{k=1}^{42} 1 = 3 \sum_{k=1}^{42} c_k - 42 = 3(45) - 42 = 93$$

7. If  $b$  is a geometric sequence in which  $b_2 = 15$  and  $b_4 = \frac{27}{5}$ , find the common ratio and find a formula for  $b_n$ :

If  $r$  is the common ratio, we know that  $b_3 = rb_2$ , and  $b_4 = rb_3$ , so  $b_4 = r^2b_2$ .  $\frac{27}{5} = 15r^2$ , so solving for  $r$  we get  $r = \frac{3}{5}$  (or 0.6 if you prefer).  $b_2 = b_1r$ , so  $15 = \frac{3}{5}b_1$ . We get  $b_1 = 25$ .

$$b_n = 25 \left(\frac{3}{5}\right)^{n-1}$$

8. Compute  $\sum_{k=1}^{\infty} 4 \left(\frac{7}{8}\right)^{k-1}$

First we note that the common ratio,  $r$ , is  $\frac{7}{8}$ . Then this is a direct application of the sum formula for infinite geometric sums:

$$\sum_{k=1}^{\infty} 4 \left(\frac{7}{8}\right)^{k-1} = \frac{4}{1 - \frac{7}{8}} = \frac{4}{\frac{1}{8}} = 4(8) = 32$$

9. Compute  $2 + \frac{5}{2} + 3 + \frac{7}{2} + 4 + \dots + 9$

Each of the terms has a common difference of  $\frac{1}{2}$ , so this is an arithmetic sum. We can write down a formula for the  $n$ th term:  $a_n = 2 + (n - 1)\frac{1}{2} = \frac{3}{2} + \frac{1}{2}n$ . Now we need to decide which term 9 is in our sum, so we substitute it's value into our formula and look for  $n$ :

$$\begin{aligned} 9 &= \frac{3}{2} + \frac{1}{2}n \\ \frac{15}{2} &= \frac{1}{2}n \\ n &= 15 \end{aligned}$$

So we can use the formula for a sum of an arithmetic sequence,

$$\frac{15}{2}(2 + 9) = 82.5$$

10. Simplify the following expression using properties of the factorial:

$$\frac{k!(k^2 + 3k + 2)}{9}$$

If we factor the polynomial it becomes clear that this is a rather silly problem:

$$\frac{k!(k^2 + 3k + 2)}{9} = \frac{k!(k + 1)(k + 2)}{9} = \frac{(k + 2)!}{9}$$

11. Write  $0.121212\overline{12}$  as a fraction

We can write the decimal as a sum,  $\sum_{n=1}^{\infty} (0.12)(0.01)^{n-1}$ . Then by the formula for infinite geometric sums,

$$\sum_{n=1}^{\infty} (0.12)(0.01)^{n-1} = \frac{0.12}{1 - 0.01} = \frac{0.12}{.99} = \frac{12}{99} = \frac{4}{33}$$