

MATH 110 - SECTION 3

Exam #2 - Sample study problems

Very important: These problems do not outline everything that will or will not be on the exam! They should be about as difficult as the exam problems and similar in flavor, however the goal of the exam is to test understanding - not regurgitation.

1. If $f(x)$ is any function, what combination of translations and flips would flip $f(x)$ horizontally, move it left 2, up 3, and shrink it to be only half it's height.

The first step is going to be decide what flips we have, since if we translated our function and then flipped it could be a mess. A horizontal flip means that the values of $f(x)$ and $f(-x)$ will switch places, so we have $f(-x)$ for our horizontal flip. "Shrinking it's height" means we have a vertical compression, so we need to multiply the output by $\frac{1}{2}$ to give us $\frac{1}{2}f(-x)$. Moving to the left 2 means we replace $f(\frac{1}{2}(-x))$ with $f(\frac{1}{2}(-x + 2))$ (the sign here is tricky, drawing a graph with a sample function can help remind us if we should add or subtract). Lastly we need to move the function up 3, so we just take the value coming out of the function and add 3, so final answer will be:

$$y = \frac{1}{2}f(-x + 2) + 3$$

2. Let $g(x) = x^2 + 3x + 2$ and let $h(x) = x^2 - 2x - 3$. Will the domain of $\left(\frac{g}{h}\right)(x)$ be the same as the domain of $\left(\frac{h}{g}\right)(x)$?

The best way to decide if the domains will be the same will be to compute each composition and then find the domain.

$$\left(\frac{g}{h}\right)(x) = \frac{x^2 + 3x + 2}{x^2 - 2x - 3}$$

Since we are interested in the domain we only want to know when something could go wrong, which since we have no square roots could only happen if we divide by zero. Factoring the bottom we get $x^2 - 2x - 3 = (x - 3)(x + 1)$, so we're dividing by zero if $x = 3$ or $x = -1$. So the domain of $\left(\frac{g}{h}\right)(x)$ is $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$. We do pretty much the same thing for $\left(\frac{h}{g}\right)(x)$,

$$\left(\frac{h}{g}\right)(x) = \frac{x^2 - 2x - 3}{x^2 + 3x + 2}$$

We need to find if we're dividing by zero, so we find when the bottom is zero:

$$x^2 + 3x + 2 = (x + 1)(x + 2)$$

So everything is ok except when $x = -1$ or $x = -2$. The domain of $\left(\frac{h}{g}\right)(x)$ is $(-\infty, -1) \cup (-1, -2) \cup (-2, \infty)$. So the domains are different.

3. If $a(x) = x^3 - \sqrt[3]{x} + \pi^2$ and $c(x) = 10 - \sqrt[3]{x} + 2\pi^2$, find a function $b(x)$ so that $(a - b)(x) = c(x)$.

This problem is basically testing if we remember definitions,

$$\begin{aligned}(a - b)(x) &= c(x) \\ a(x) - b(x) &= c(x) \\ x^3 - \sqrt[3]{x} + \pi^2 - b(x) &= 10 - \sqrt[3]{x} + 2\pi^2 \\ -b(x) &= 10 - \sqrt[3]{x} + 2\pi^2 - (x^3 - \sqrt[3]{x} + \pi^2) \\ b(x) &= -10 + \sqrt[3]{x} - 2\pi^2 + (x^3 - \sqrt[3]{x} + \pi^2) \\ b(x) &= -10 + x^3 - \pi^2\end{aligned}$$

It's always a good idea to double check our answer,

$$\begin{aligned}a(x) - b(x) &= x^3 - \sqrt[3]{x} + \pi^2 - (-10 + x^3 - \pi^2) \\ &= x^3 - \sqrt[3]{x} + \pi^2 + 10 - x^3 + \pi^2 \\ &= 10 - \sqrt[3]{x} + 2\pi^2 \\ &= c(x)\end{aligned}$$

4. If $f(x) = |x| + 5$ and $g(x) = |x| + 3$, will $(f \circ g)(x) = (g \circ f)(x)$?

Problems like this can be misleading since we need to remember that $(f \circ g)(x)$ is NOT multiplication. The easiest way with any question about composition is to write out the composition then answer the question.

$$\begin{aligned}(f \circ g)(x) &= f(|x| + 3) \\ &= ||x| + 3| + 5 \\ &= |x| + 8\end{aligned}$$

We have the triple check that what we did in the last step is ok, since absolute values are tricky, but in this particular case $|x|$ will always be non-negative and 3 is also non-negative, so we know we're adding two positive numbers so we can drop the outside

absolute value (being careless with absolute value signs is a prime way to make algebra mistakes!).

$$\begin{aligned}(g \circ f)(x) &= g(|x| + 5) \\ &= ||x| + 5| + 3 \\ &= |x| + 8\end{aligned}$$

So in this case the composition is the same, but only because of the specific details. If you pick two random functions the odds are very high that $(f \circ g)(x) \neq (g \circ f)(x)$.

5. Write $j(x) = (2x - 9)^6 + 11$ as a composition of two functions (don't use the trivial solution).

The outside takes a value and returns that value to the sixth power plus 11, so this suggests a function. Let $k(x) = x^6 + 11$ for the outside and $l(x) = 2x - 9$ for the inside. We need to see if our solution works,

$$(k \circ l) = k(2x - 9) = (2x - 9)^6 + 11$$

So $j(x) = (k \circ l)(x)$.

6. Write the equation for the line parallel to $h(x) = 2x - 6$ which passes through $(1, 6)$.

If two lines are parallel they have the same slope, so the slope of our line is 2. Our line has the form $y = 2x + b$ for some b , so we plug in the values of the point to find b :

$$\begin{aligned}y &= 2x + b \\ 6 &= 2(1) + b \\ 4 &= b\end{aligned}$$

So the line in question is $y = 2x + 4$

7. Rewrite $f(x) = 2x^2 - 12x + 7$ in standard form and describe the vertex.

We can use completing the square to write $f(x)$ in standard form,

$$\begin{aligned}f(x) &= 2x^2 - 12x + 7 \\ &= 2(x^2 - 6x) + 7 \\ &= 2(x^2 - 6x + 9 - 9) + 7 \\ &= 2(x^2 - 6x + 9) - 9(2) + 7 \\ &= 2(x - 3)^2 - 11\end{aligned}$$

The vertex will be at $(3, -11)$. The question is a bit vague, but we can say that since the leading coefficient (the 2 on the $2x^2$) is positive the vertex will be a minimum.

9. Construct a degree 4 polynomial $h(x)$ such that $h(2) = 0$, $h(-4) = 0$, $h(-1) = 0$ and $h(-\frac{1}{2}) = 0$. Additionally, $h(x)$ must pass through the point $(1, 45)$

We can write down a general form of $h(x)$ based on it's zeros: $h(x) = a(x - 2)(x + 4)(x + 1)(x + \frac{1}{2})$ for some number a . Note that the sign of the zeros changes since we need the $(x - 2)$ factor to be zero at $x = 2$, etc. The problem calls for a degree 4 polynomial and we already have 4 factors, so all that remains is to find the correct a . If we plug in the values from our point,

$$\begin{aligned} h(x) &= a(x - 2)(x + 4)(x + 1)(x + \frac{1}{2}) \\ 45 &= a(1 - 2)(1 + 4)(1 + 1)(1 + \frac{1}{2}) \\ 45 &= a(-1)(5)(2)(\frac{3}{2}) \\ 45 &= a(-15) \\ -3 &= a \end{aligned}$$

So our polynomial is $h(x) = -3(x - 2)(x + 4)(x + 1)(x + \frac{1}{2})$ and $h(1) = 45$ as required. Factored form is usually more useful than writing out all the terms, but if need to we can do the algebra to get $h(x) = -2x^4 - 0.7x^3 + .9x^2 + 22x + 8$ (see, wasn't factored form nicer?).

10. The Kelvin scale is a way of measuring temperature from absolute zero (there is actually a temperature which can't get any colder!). If 0 degrees kelvin is equal to -459 Fahrenheit, and water freezes at 273 kelvin (that's 32 degree Fahrenheit), write a linear function for the temperature in Kelvin as function of degrees Fahrenheit.

We want a function which will take a temperature in Kelvin and spit out a temperature in Fahrenheit, so x will be the temperature in Kelvin and $f(x)$ will be temperature in Fahrenheit. We have two points of data: $(0, -459)$ and $(273, 32)$. The first step to fitting a line is always finding the slope,

$$m = \frac{\Delta Y}{\Delta X} = \frac{32 - (-459)}{273 - 0} \approx 1.798$$

So $f(x) = 1.798x + b$ for some b . If we pick a point we can find b ,

$$-459 = 1.798(0) + b$$

So $f(x) = 1.798x - 459$

You should review all of sections 1.5, 1.6, 2.1, 2.2, 3.1, and 3.2, since anything in those sections is fair game for the exam.