Math 110 - Section 3
Exam #1 - Sample problem solutions

Very important: These problems do not outline everything that will or will not be on the exam! They should be about as difficult as the exam problems and similar in flavor, however the goal of the exam is to test understanding - not regurgitation.

1. Recall the definition of a function: Something is a function assigns some value to each input.

   (a) Explain what the vertical line test is and how to use it.
   The vertical line test says that if we draw a vertical line on our on our graph it should never intersect the function at two or more points. We can use the vertical line test by imagining we drag the line across our graph looking for places where it intersects the function twice.

   (b) Explain what it means if something fails the vertical line test.
   If a graph fails the vertical line test it means that for some value of x there are two values of y, which can't happen if we have a function.

   (c) If an equation is undefined for some values, can it still potentially pass the vertical line test?
   Yes. The vertical line test only says that we're never allowed to get two values for any given x, it is alright if the function has no value there (ie, it is undefined).

2. For a piecewise function,

$$j(x) = \begin{cases} \sqrt{-2-x} & \text{if } x < 0 \\ x^3 - 2\sqrt{2x} & \text{if } 0 \leq x \leq 1 \\ \pi & \text{if } x > 1 \end{cases}$$

   (a) Find $j(-4)$:
   Remember: When we're evaluating a piecewise function, we're looking at the value of x to decide which rule to use, not the value we get back.
   $j(-4) = \sqrt{-2-4} = \sqrt{2}$

   (b) Find $j(-1)$:
   $j(-1) = \sqrt{-2-1} = \sqrt{-1}$ *which is undefined*

   (c) Find $j(1)$:
   $j(1) = 1^3 - 2\sqrt{2*1} = 1 - 2\sqrt{2}$ *This is as much as we can simplify since we know that the $\sqrt{2}$ is irrational*

   (d) Find $j(5)$:
   We use the bottom rule, so $j(5) = \pi$.

3. Evaluate the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = x^2 + x - 2$

   The key to this problem is to handle the $f(x+h)$ part correctly, and the rest will follow.

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{((x+h)^2+(x+h)-2)-(x^2+x-2)}{h} \\
&= \frac{(x^2+2hx+h^2+x+h-2)-(x^2+x-2)}{h} \\
&= \frac{x^2+2hx+h^2+x+h-2-x^2-x+2}{h} \\
&= \frac{h^2+2hx+h}{h} \\
&= h+2x+1
\end{aligned}$$

4. Find the zeros and any places where the function might be undefined for

$$g(x) = \frac{x^2-3x+2}{x^2+7x+12}$$

   Use this information to write the domain of $g(x)$.

   If we set $g(x)$ equal to zero we get

$$\begin{aligned}
0 &= \frac{x^2-3x+2}{x^2+7x+12} && \textit{Multiplying both sides by the bottom we get} \\
0 &= x^2-3x+2 && \textit{Now we factor the equation} \\
0 &= (x-1)(x-2) && \textit{So we have two solutions} \\
x &= 1, \qquad x = 2
\end{aligned}$$

   So $g(1) = 0$ and $g(2) = 0$.

   For this function to be undefined the bottom piece would have to be zero, so we set it to zero to find out when that can happen:

$$\begin{aligned}
0 &= x^2+7x+12 && \textit{Now we factor...} \\
&= (x+3)(x+4) \\
x &= -3, \qquad x = -4
\end{aligned}$$

   So $g(x)$ is defined everywhere but at $x = -3$ and $x = -4$. So the domain of $g(x)$ is $(-\infty,-4)\bigcup(-4,-3)\bigcup(-3,\infty)$

5. Give an example of an equation whose solution is not a function
   $y^2 = x$ will not produce a function, since when we try to solve for y by taking the square root we get $y = \pm\sqrt{x}$ which means that we get two values of y for each value of x.

6. Draw a graph of the function with your calculator and show your window

$$p(x) = 0.2x^3 - 6x^2 - 4x$$

Graph not shown here, but you can see the function with the following window:

$$Xmin = -80 \qquad Ymin = -1000$$
$$Xmax = 80 \qquad Ymax = 1000$$

7. Find the intervals on which the function from the previous problem is negative and the intervals on which it is decreasing

The function is negative on all values less than $x \approx 30.6$, so the interval would be approximately $(-\infty, 30.6)$. However, just because it's negative doesn't mean it's not increasing, in fact the only place it's decreasing is from 0 to approximately $x = 20.3$, so the interval would be $(0, 20.3)$. (For finding the minimum and the zero we could use trace or the Calc feature to get a more exact answer)

8. Is $f(x) = x^4 + 9$ even or odd?
Try substituting in $-x$ to see what we get: $f(-x) = (-x)^4 + 9 = x^4 + 9$. So $f(x) = f(-x)$, which means that the function is even.

9. A parking lot is being put up next to a building, using the building for one side of the parking lot. If the builders only have 250 ft. of fencing, what dimensions will give them the biggest parking lot?

The area of the parking lot will be the length times the width, so $A = lw$. The problem doesn't tell us which side of the parking lot the building is on, so we can assume it goes along the length of one side (the problem just asks for the final dimensions). So the perimeter is (width) + (width) + (length). In math, $2w + l = 250$. Solving the equation for l we get $l = 250 - 2w$. We can plug this into our original equation for l to get $A = (250 - 2w)w$.

Using a graphing calculator and the ZoomFit function in the zoom menu we get a window that cuts off the top part of the graph, so by going into the Window menu we can increase increase the Ymin to 0 (since we only want the maximum and it's above the axis) and the Ymax 10% or so. Going back to Graph the function looks a little better, now we can use the Maximum function in the Calc menu to find the maximum: $x = 62.5$. This tells us one side of the parking lot, now going back to the equation for perimeter (remember we used x for w) we find the length, $l = 250 - 2(62.5) = 125$.

So our answer is 62.5 ft by 125 ft.

10. Happy trails coffee supplies charges a signing fee of \$2000 upon establishing a contract and each pound of coffee costs \$1.35. Joe's Roasting Co charges \$3.10 per pound of coffee, but no signing fee. When does Happy trails become a better deal?

Let $H(x)$ be the cost of buying x pounds of coffee from Happy trails and $J(x)$ be the cost of buying x pounds from Joe's. Happy trails will charge \$2000 *to setup the contract even if we never buy a coffee bean from them, so the cost of buying from Happy trails will be (pounds of coffee)(price per pound)+*\$2000*. Writing all this out,* $H(x) = 1.35x + 2000$.

The function for Joe's is a little easier since they charge a flat fee per pound, so $J(x) = 3.10x$.

The problem is asking for how many pounds of coffee we would have to buy to get the same price from either vendor, we could either solve the problem by putting both equations into a graphing calculator and finding the intersection, or setting them equal to each other and solving. Suppose we do it the second way:

$1.35x + 2000 = 3.10x$. *Subtracting* $1.35x$ *from both sides we get* $2000 = 1.75x$. *So* $x \approx 1142$ *pounds of coffee (that's a lot of coffee!).*

11. Can you find a function which is even and odd?
Suppose $f(x)$ is even and odd. Even means that for every x we have $f(-x) = f(x)$, and odd means that $f(-x) = -f(x)$. We can combine the equations to see that $f(x) = -f(x)$. At this point we might be able to guess the answer, but continuing with the algebra we get $2f(x) = 0$, which means that $f(x)$ must always be zero. So the function which is always zero is even and odd.