MATH 464:
TEST 2
MAKE UP

SPRING 2016

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

Rules to the Make-Up:

Here are the rules to the make-up. You have two choices. Either you turn in the make-up, or you do not. If you turn in the make-up, it will be graded, and you will receive a new grade for Exam 2. That new grade will be the average of your grade on Exam 2 and the Make-Up to Exam 2. If you do not do the Make-Up, your grade on Exam 2 remains the same. The Make-Up is due on **Tuesday, April 26, 2016**.

Do all of the following problems. An answer alone will receive no credit. Justify all your claims.

(1) Roll a fair, 4-sided die N times where N is a Poisson random variable with parameter $\lambda > 0$. Let X be the number of 3's rolled in this experiment. Find $E(X)$.

(2) You are dealt 5 cards from a standard deck. You keep careful track of the order of the cards you are dealt. For both questions below, give an exact answer and a decimal approximation accurate to the nearest one-hundredth.

a) What is the probability that you get one ace and the rest are face cards? (Face cards are the jack, the queen, and the king.)

b) What is the probability that you have a pair of tens and a pair of threes (and no better)?

(3) I have 30 books. 5 are labeled classics, 10 are labeled mysteries, 7 are labeled science, and the rest are sports. If I randomly select 6 books, what is the probability I

a) select at least 2 science books?

b) only select books that are mysteries or sports books?

c) select books with precisely 2 being classics and 1 being science.

(4) Consider an experiment where you roll two fair, 4-sided dice. Label one as die 1 and one as die 2. Let X be the random variable which is the sum of the values on die 1 and die 2. Let Y be a random variable which is the value of die 1 minus the value of die 2.

a) Find the pmfs for X and Y individually. Write them as tables.

b) Find the joint pmf of X and Y. Write it as a table. Are X and Y independent? Explain.

c) Find $E(XY)$.

(5) Let X and Y be independent, discrete random variables. Suppose that

$$f_X(k) = f_Y(k) = p(1-p)^k \quad \text{for all } k = 0, 1, 2, \cdots$$

for some $0 < p < 1$. Show that for any $n \geq 0$,

$$P(X = k \,|\, X + Y = n) = \frac{1}{n+1} \quad \text{for any } 0 \leq k \leq n.$$

(6) a) Let X be a continuous random variable with pdf

$$f_X(t) = \exp[-t - \exp(-t)] \quad \text{for all } t \in \mathbb{R}\,.$$

Find $F_X(x)$.

b) Find the real number a for which

$$f_X(x) = \begin{cases} a(x+1) & -1 \le x \le 0 \\ a(x-1)^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

is the probability density function for a continuous random variable X.

(7) Let X be a continuous random variable with uniform distribution on $[0, 1]$. Let Y be a continuous random variable with uniform distribution on $[0, 2]$. Suppose X and Y are independent and consider the new random variable $Z = X + Y$.

a) Find the cdf of Z. **Hint:** It may be helpful to draw some pictures.

b) Find the pdf of Z.

c) Find the mean of Z using part b). Is this the right answer?