MATH 464
HOMEWORK 9

SPRING 2016

The following assignment is to be turned in on
Thursday, April 28, 2016.

1. Let X be a normal random variable with parameters $\mu \in \mathbb{R}$ and $\sigma > 0$.
Find the cdf and pdf for Z the standardization of X.

2. Let X_1, X_2, $\cdots$, X_n be independent, identically distributed (i.i.d) random
variables. Let μ and σ^2 be the common mean and variance respectively.
Define
$$Y = \frac{1}{n-1}\sum_{j=1}^{n}(X_j - \overline{X}_n)^2 \quad \text{where} \quad \overline{X}_n = \frac{1}{n}\sum_{j=1}^{n}X_j$$
Find the mean of Y in terms of n, μ, and σ^2.

3. Flip a fair coin until you get 100 heads. Use the central limit theorem
to find (approximately) the probabilities it takes at least 200, 250, and 300
flips. **Hint:** Let X be the number of flips to get 100 heads. Write X as the
sum of 100 i.i.d. random variables.

4. Let X_1, X_2, $\cdots$, X_n be independent random variables each having the
standard normal distribution.

a) Find (approximately)
$$P\left(80 \leq \sum_{j=1}^{100} X_j^2 \leq 120\right)$$

b) Find c so that
$$P\left(\left|\sum_{j=1}^{100} X_j^2 - 100\right| < c\right) = 0.95\,.$$