MATH 464
HOMEWORK 8

SPRING 2016

The following assignment is to be turned in on
Thursday, April 7, 2016.

1. Let X be an exponential random variable with parameter $\lambda > 0$.

a) Let $t \geq 0$ and calculate $P(X \geq t)$.

b) Let $s, t \geq 0$ and calculate $P(X \geq s + t | X \geq s)$. (You can compare your answer to this question with your answer to problem #5 on homework #5.)

2. Let X be a normally distributed random variable with real parameters μ and $\sigma > 0$. Find the mean and variance of X. **Hint:** It may be useful to remember that

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}\,.$$

3. The gamma function is defined by

$$\Gamma(w) = \int_0^{\infty} x^{w-1} e^{-x}\, dx$$

for all $w > 0$. In terms of this function, a continuous random variable X (with parameters $w > 0$ and $\lambda > 0$) is defined by setting

$$f_X(x) = \begin{cases} \frac{\lambda^w}{\Gamma(w)} x^{w-1} e^{-\lambda x} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

and declaring that X has probability density function $f_X(x)$. (f_X is called the gamma distribution with parameters $w > 0$ and $\lambda > 0$.)

a) Show that X is a continuous random variable by showing that

$$\int_{\mathbb{R}} f_X(t)\, dt = 1$$

for all values of $w > 0$ and $\lambda > 0$.

b) Show that for any $w > 1$,

$$\Gamma(w) = (w-1)\Gamma(w-1)$$

Use your result to calculate $\Gamma(n)$ for any integer $n \geq 2$.

c) Compute the mean and variance of this random variable X.

4. Let X be uniformly distributed on $[0,1]$. Find the cumulative distribution function (i.e. the cdf) for the random variable

$$Y = \frac{3X}{1-X}$$