MATH 464
HOMEWORK 7

SPRING 2016

The following assignment is to be turned in on
Thursday, March 31, 2016.

1. Consider the following experiment: Roll 2 fair, four sided dice. Consider the following discrete random variables:

$X =$ the number of odd dice.

$Y =$ the number of even dice.

$Z =$ the number of dice showing a 3 or a 4.

Clearly, each of X, Y, and Z have range $\{0, 1, 2\}$.

a) Find $f_{X,Y}(x, y)$. Give your answer in tabular form.

b) Determine whether or not X and Y are independent.

c) Find $E(XY)$.

d) Repeat exercises a) - c) above for the random variables Y and Z.

2. Suppose that X and Y are discrete random variables and that you know the joint probability mass function of X and Y is:

$$f_{X,Y}(x, y) = \alpha^{x+y+1} \quad \text{for } x, y = 0, 1, 2 \quad \text{with some } \alpha > 0.$$

Find $E(XY)$ and $E(Y)$.

3. Let X and Y be independent discrete random variables. Suppose we know that

$$E(X) = -2, \quad E(X^2) = 5, \quad E(X^3) = 10, \quad \text{and} \quad E(X^4) = 50$$

and

$$E(Y) = -1, \quad E(Y^2) = 5, \quad E(Y^3) = -13, \quad \text{and} \quad E(Y^4) = 73$$

a) Let $Z = 2X + Y$. Find the mean and variance of Z.

b) Let $W = Y^2 - 2YX^2$. Find the mean and variance of W.

4. Let X and Y be independent discrete random variables. Suppose X is a Poisson random variable with parameter $\lambda > 0$ and Y is a Poisson random variable with parameter $\mu > 0$. Show that the random variable $Z = X + Y$ is also a Poisson random variable and determine its parameter. **Hint:** You

may want to use the formula:

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k \quad \text{for any integer } n \geq 1 \quad \text{and real number } x.$$

5. Suppose you have an unfair coin with probability p for heads. Consider the following 2 stage experiment: First, flip the coin until you get a heads. Then, flip the coin again until you get a tails. Let X be the discrete random variable counting the total number of flips in this 2 stage experiment.

a) Find the mean and variance of X. **Hint:** It may be useful to write X as the sum of 2 random variables. If you do, label and describe carefully each of these random variables.

b) Let Y be the number of heads minus the number of tails in this 2 stage experiment. Find the mean and variance of Y.

6. Let X and Y be independent discrete random variables. Suppose that each of them is geometric and that you know $E(X) = 2$ and $E(Y) = 3$.

a) Find the joint probability mass function of X and Y.

b) Find the probability that $X + Y \leq 4$.

c) Consider $W = \min\{X, Y\}$ and $Z = \max\{X, Y\}$. Find the joint probability mass function of W and Z.

7. Let $X_1, X_2, \cdots, X_{100}$ be independent discrete random variables. Suppose that each of them is a Poisson random variable with $\lambda = 2$. Consider

$$\overline{X} = \frac{1}{100} \sum_{j=1}^{100} X_j$$

which is sometimes called the *sample mean*. Find the mean and variance of $\overline{X}$.

8. Suppose you have an unfair coin with probability p for heads. Do an experiment where you flip this coin N times, and let N be a random number which is Poisson with parameter $\lambda > 0$. Assume that N is independent of the outcomes of the flips. Let X be the number of heads. Let Y be the number of tails. Find the probability mass functions for X and Y and use your result to show that X and Y are independent.