MATH 464
HOMEWORK 6

SPRING 2016

The following assignment is to be turned in on
Thursday, March 10, 2016.

1. Let X be the number of eggs laid by an insect. Suppose that X is a Poisson random variable with parameter $\lambda > 0$. Suppose that each egg produces an insect with probability $0 < p < 1$, and assume that the eggs are independent of each other. Let Y be the number of insects that hatch from the X eggs.

a) Find $E(Y)$.

b) Show that Y is also a Poisson random variable by calculating its parameter (in terms of λ and p).

Hint: For both parts above, use the partition theorem with $B_k = \{\omega \in \Omega : X(\omega) = k\}$ with $k \geq 0$. Note also: If we are given that $X = k$, then Y is a binomial random variable.

2. Suppose that in a certain state the license plates have three letters followed by 3 numbers. If no letter or number can be repeated, how many license plates are possible?

3. A club has 50 members. The club needs to form two committees, one with 8 members and one with 7 members. How many ways can this be done if no one is allowed to serve on two committees at the same time?

4. 6 students, 3 boys and 3 girls, line up in random order for a photograph. What is the probability that the boys and girls alternate?

5. A fair coin is tossed 10 times. What is the probability of exactly 5 heads? What is the probability of at least 5 heads?

6. I have a television with 50 channels. On a certain evening, 12 channels are showing sit-coms, 17 are showing reality shows, 15 are showing movies, and the remaining 6 are showing something else. If I randomly pick 5 of the channels and look at what is showing, what is the probability that I see:

a) exactly 2 movies, 1 sit-com, and 2 reality shows?

b) at least one movie?

c) only sit-coms and reality shows?

7. Consider a usual deck of cards. Draw five cards at random. What is the probability you get:

a) "four of a kind" or four cards of the same rank?

b) a "full-house" or three cards of the same rank and two cards of the same rank?

c) "three of a kind" or three cards of the same rank, but you do not have a "full-house" or "four of a kind"?