MATH 464
HOMEWORK 4

SPRING 2016

The following assignment is to be turned in on **Thursday, February 18, 2016**.

1. Consider an experiment where you roll a fair 4-sided die twice. Let X be the discrete random variable corresponding to the sum of the rolls. Let $Y = X^2 - 4$. Find the pmf's, i.e., f_X and f_Y.

2. Let $0 < p \leq 1$ and consider a function X with range $\{1, 2, 3, \cdots\}$ and corresponding numbers

$$P(X = k) = p(1-p)^{k-1} \quad \text{for any integer } k \geq 1\,.$$

Prove that X is a discrete random variable by showing that the sum of the above probabilities is 1. This is the geometric random variable with parameter p.

3. Let X be a Poisson random variable with parameter $\lambda > 0$. Compute the following:

a) $P(2 \leq X \leq 4)$
b) $P(X \geq 5)$
c) $P(X \text{ is even})$

give each answer in exact form and, with the choice of $\lambda = 2$, give a decimal approximation to the above which is accurate to 3 decimal places.

4. Let X be a discrete random variable whose range is $\{0, 1, 2, 3, \cdots\}$. Prove that

$$E(X) = \sum_{k=0}^{\infty} P(X > k)\,.$$

5. Compute the expected value of the geometric random variable with parameter $0 < p \leq 1$. **Hint:** Use problem 4 above.

6. Let X be a binomial random variable with parameters $0 \leq p \leq 1$ and $n > 0$ an integer. For any $0 \leq k \leq n$, denote by $P_k = P(X = k)$. Compute

the ratio

$$\frac{P_{k-1}}{P_k} \quad \text{for } 1 \leq k \leq n\,.$$

Show that this ratio is less than one if and only if $k < np + p$. This shows that the most probable values of X are those near np.

7. Let X be a Poisson random variable with parameter $\lambda > 0$. Let $g : \mathbb{R} \to \mathbb{R}$ be the function $g(x) = x(x-1)$. Set $Y = g(X)$. Find $E(Y)$.

8. Let X be a function whose range is $\{1, 2, 3, \cdots\}$. Consider the values

$$P(X = n) = \frac{1}{n(n+1)} \quad \text{for any } n \geq 1\,.$$

Does this function X define a discrete random variable? If so, what is $E(X)$?