MATH 464
HOMEWORK 2

SPRING 2016

The following assignment is to be turned in on
Thursday, February 4, 2016.

1. Suppose we pick a letter at random from the word MISSISSIPPI. Write down a sample space and give the probability of each outcome?

2. In a group of students, 25% smoke, 60% drink, and 15% do both. What percentage of the students that either smoke or drink?

3. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Suppose $A, B \in \mathcal{F}$ with:

$$P(A) = \frac{1}{3} \quad P(A^c \cap B^c) = \frac{1}{2} \quad \text{and} \quad P(A \cap B) = \frac{1}{4}\,.$$

What is $P(B)$?

4. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Suppose $A, B \in \mathcal{F}$ with $P(A) = 0.4$ and $P(B) = 0.7$. What are the maximum and minimum possible values for $P(A \cap B)$?

5. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let $A, B, C \in \mathcal{F}$. Prove that

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)\,.$$

This is a special case of a more general result called the inclusion-exculsion formula.

6. Alice and Bob take turns flipping a fair coin. The game is: the first one to heads wins. Bob lets Alice flip first. What is the probability that she wins?

7. An unfair coin has probability 1/3 for heads and 2/3 for tails. Do an experiment where you flip this coin until you get heads and then stop. What is the probability it takes exactly 8 flips, given that it takes at least 6 flips?

8. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let $A, B \in \mathcal{F}$ with $P(B) > 0$. Prove that

$$P(A^c|B) = 1 - P(A|B)\,.$$

using the definition of conditional probability measures. Do not use the fact that conditional probabilities define probability measures.

9. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Suppose $A, B \in \mathcal{F}$ are independent events. Prove that A^c and B^c are independent events.

10. Roll a fair, six-sided die twice. Let A be the event that the first roll is odd. Let B be the event that the second roll is even. Let C be the event that either both rolls are even or both rolls are odd. Show that A, B, and C are pairwise independent, but not independent.